

CONSTRUCTION MANAGEMENT
Concept-Develop-Execute-Finish

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Concrete Structures
(ACI 318-05)
in Mathcad

Sixth Edition

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1. Unit Conversion

Length

in = inch

ft = foot

yd = yard

mi = mile

1 in = 25.4 · mm

1 cm = 0.394 · in

1 ft = 0.305 m

1 m = 3.281 · ft

1 ft = 12 · in

1 yd = 0.914 m

1 m = 1.094 · yd

1 yd = 3 · ft

1 mi = 1.609 · km

1 km = 0.621 · mi

1 mi = 1760 · yd

1 mi = 5280 · ft

1 m + 100 mm = 1.1 m

1 mi + 200 m = 1.124 · mi

Size of standard concrete cylinder

D := 6 in = 15.24 · cm

H := 12 in = 30.48 · cm

Force

lbf = pound force

kip = kilopound force

1 lbf = 4.448 N

1 lbf = 0.454 · kgf

1 kgf = 9.807 N

1 N = 0.225 · lbf

1 kgf = 2.205 · lbf

1 N = 0.102 · kgf

1 kip = 4.448 · kN

1 kip = 0.454 · tonnef

1 tonf = 0.907 · tonnef

1 kN = 0.225 · kip

1 tonnef = 2.205 · kip

1 tonnef = 1.102 · tonf

1 tonnef = 1000 · kgf

1 tonf = 2000 · lbf

Stress

psi = pound per square inch

$$1\text{psi} = 1 \cdot \frac{\text{lbf}}{\text{in}^2}$$

ksi = kilopound per square inch

$$1\text{ksi} = 1 \cdot \frac{\text{kip}}{\text{in}^2}$$

psf = pound per square foot

$$1\text{psf} = 1 \cdot \frac{\text{lbf}}{\text{ft}^2}$$

$$1\text{psi} = 6.895 \cdot \text{kPa}$$

$$1\text{kPa} = 0.145 \cdot \text{psi}$$

$$1\text{Pa} = 1 \cdot \frac{\text{N}}{\text{m}^2}$$

$$1\text{ksi} = 6.895 \cdot \text{MPa}$$

$$1\text{MPa} = 0.145 \cdot \text{ksi}$$

$$1\text{kPa} = 1 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$1\text{MPa} = 1 \cdot \frac{\text{N}}{\text{mm}^2}$$

$$1\text{psf} = 0.048 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$1 \frac{\text{kN}}{\text{m}^2} = 20.885 \cdot \text{psf}$$

$$1\text{psf} = 4.882 \cdot \frac{\text{kgf}}{\text{m}^2}$$

$$1 \frac{\text{kgf}}{\text{m}^2} = 0.205 \cdot \text{psf}$$

Concrete compression strength

$$3000\text{psi} = 20.7 \cdot \text{MPa}$$

$$20\text{MPa} = 2900.8 \cdot \text{psi}$$

$$4000\text{psi} = 27.6 \cdot \text{MPa}$$

$$25\text{MPa} = 3625.9 \cdot \text{psi}$$

$$5000\text{psi} = 34.5 \cdot \text{MPa}$$

$$35\text{MPa} = 5076.3 \cdot \text{psi}$$

$$8000\text{psi} = 55.2 \cdot \text{MPa}$$

$$55\text{MPa} = 7977.1 \cdot \text{psi}$$

Steel yield strength

$$60\text{ksi} = 413.7 \cdot \text{MPa}$$

$$390\text{MPa} = 56.6 \cdot \text{ksi}$$

$$75\text{ksi} = 517.1 \cdot \text{MPa}$$

$$490\text{MPa} = 71.1 \cdot \text{ksi}$$

Live loads

$$40\text{psf} = 1.915 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$200 \frac{\text{kgf}}{\text{m}^2} = 40.963 \cdot \text{psf}$$

$$100\text{psf} = 4.788 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$4.80 \frac{\text{kN}}{\text{m}^2} = 100.25 \cdot \text{psf}$$

Density

$$1\text{pcf} = 1 \cdot \frac{\text{lbf}}{\text{ft}^3}$$

$$125\text{pcf} = 19.636 \cdot \frac{\text{kN}}{\text{m}^3}$$

$$145\text{pcf} = 22.778 \cdot \frac{\text{kN}}{\text{m}^3}$$

Moments

$$1\text{ft} \cdot \text{kip} = 1.356 \cdot \text{kN} \cdot \text{m}$$

User setting

$$\text{Riels} := 1$$

$$\text{USD} := 4165\text{Riels}$$

$$124\text{USD} = 516460 \cdot \text{Riels}$$

$$200000\text{Riels} = 48.019 \cdot \text{USD}$$

$$\text{Cost}_{\text{Steel}} := 665 \frac{\text{USD}}{1\text{tonnef}}$$

$$670\text{kgf} \cdot \text{Cost}_{\text{Steel}} = 445.55 \cdot \text{USD}$$

2. Simple Calculation

$$\left(1 + \sqrt{3 + \frac{\sqrt{2}}{3}}\right)^3 = 23.472$$

$$a := 2 \qquad b := 4 \qquad c := -1$$

$$\Delta := b^2 - 4 \cdot a \cdot c$$

$$x_1 := \frac{-b + \sqrt{\Delta}}{2 \cdot a} = 0.225$$

$$x_2 := \frac{-b - \sqrt{\Delta}}{2 \cdot a} = -2.225$$

3. Materials

1. Concrete

Standard concrete cylinder

$$D := 6\text{in} = 15.24\cdot\text{cm}$$

$$D := 150\text{mm}$$

$$H := 12\text{in} = 30.48\cdot\text{cm}$$

$$H := 300\text{mm}$$

Concrete compression strength

$$f'_c := 3000\text{psi} = 20.7\cdot\text{MPa}$$

$$f'_c := 20\text{MPa} = 2900.8\cdot\text{psi}$$

$$f'_c := 4000\text{psi} = 27.6\cdot\text{MPa}$$

$$f'_c := 25\text{MPa} = 3625.9\cdot\text{psi}$$

$$f'_c := 5000\text{psi} = 34.5\cdot\text{MPa}$$

$$f'_c := 35\text{MPa} = 5076.3\cdot\text{psi}$$

Concrete ultimate strain

$$\epsilon_u := 0.003$$

Cubic and cylinder compression strength

$$f_{\text{cube}} = \frac{f'_c}{0.85}$$

$$f'_c := 20\text{MPa}$$

$$f_{\text{cube}} := \frac{f'_c}{0.85} = 23.529\cdot\text{MPa}$$

$$f'_c := 25\text{MPa}$$

$$f_{\text{cube}} := \frac{f'_c}{0.85} = 29.412\cdot\text{MPa}$$

$$f'_c := 35\text{MPa}$$

$$f_{\text{cube}} := \frac{f'_c}{0.85} = 41.176\cdot\text{MPa}$$

Modulus of rupture (tensile strength)

$$f_T = 7.5\cdot\sqrt{f'_c} \quad (\text{in psi})$$

Metric coefficient

$$7.5\text{psi}\cdot\sqrt{\frac{f'_c}{\text{psi}}} = 7.5\text{MPa}\cdot\frac{\text{psi}}{\text{MPa}}\cdot\sqrt{\frac{f'_c}{\text{MPa}}\cdot\frac{\text{MPa}}{\text{psi}}} = C\cdot\text{MPa}\cdot\sqrt{\frac{f'_c}{\text{MPa}}}$$

$$C := 7.5\frac{\text{psi}}{\text{MPa}}\cdot\sqrt{\frac{\text{MPa}}{\text{psi}}} = 0.623$$

$$f_T = 0.623\cdot\sqrt{f'_c} \quad (\text{in MPa})$$

Modulus of elasticity

$$E_c = 33 \cdot w_c^{1.5} \cdot \sqrt{f'_c} \quad (\text{in psi})$$

w_c is a unit weight of concrete (in pcf)

$$\text{Metric coefficient} \quad 33 \text{psi} \cdot \left(\frac{\frac{\text{kN}}{\text{m}^3}}{\text{pcf}} \right)^{1.5} \cdot \sqrt{\frac{\text{MPa}}{\text{psi}}} = 44.011 \cdot \text{MPa}$$

$$E_c = 44 \cdot w_c^{1.5} \cdot \sqrt{f'_c} \quad (\text{in MPa})$$

w_c in kN/m^3

Example 3.1

Concrete compression strength $f'_c := 25 \text{MPa}$

Unit weight of concrete $w_c := 24 \frac{\text{kN}}{\text{m}^3} \quad 145 \text{pcf} = 22.778 \cdot \frac{\text{kN}}{\text{m}^3}$

Modulus of rupture

$$f_r := 7.5 \text{psi} \cdot \sqrt{\frac{f'_c}{\text{psi}}} = 3.114 \cdot \text{MPa}$$

$$f_r := 0.623 \text{MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}} = 3.115 \cdot \text{MPa}$$

Modulus of elasticity

$$E_c := 33 \text{psi} \cdot \left(\frac{w_c}{\text{pcf}} \right)^{1.5} \cdot \sqrt{\frac{f'_c}{\text{psi}}} = 2.587 \times 10^4 \cdot \text{MPa}$$

$$E_c := 44 \text{MPa} \cdot \left(\frac{\frac{w_c}{\text{kN}}}{\frac{\text{m}^3}}{\text{m}^3}} \right)^{1.5} \cdot \sqrt{\frac{f'_c}{\text{MPa}}} = 2.587 \times 10^4 \cdot \text{MPa}$$

2. Steel Reinforcements

Steel yield strength of deformed bar (DB)

$$f_y := 390\text{MPa} = 56.565\text{·ksi}$$

Steel yield strength of round bar (RB)

$$f_y := 235\text{MPa} = 34.084\text{·ksi}$$

Modulus of elasticity

$$E_s := 29000000\text{psi} = 1.999 \times 10^5 \cdot \text{MPa}$$

$$E_s := 2 \cdot 10^5 \text{MPa}$$

Steel yield strength

$$\varepsilon_y = \frac{f_y}{E_s}$$

US Steel Reinforcements

Bar No. (#)

Bar diameter

$$\text{no} := \begin{pmatrix} 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \end{pmatrix}$$

$$D := \frac{\text{no}}{8} \text{in}$$

$$D = \begin{pmatrix} 9.5 \\ 12.7 \\ 15.9 \\ 19 \\ 22.2 \\ 25.4 \\ 28.6 \\ 31.8 \\ 34.9 \\ 38.1 \\ 41.3 \\ 44.4 \end{pmatrix} \cdot \text{mm}$$

Steel area

$$A := \frac{\pi \cdot D^2}{4}$$

$$A = \begin{pmatrix} 0.71 \\ 1.27 \\ 1.98 \\ 2.85 \\ 3.88 \\ 5.07 \\ 6.41 \\ 7.92 \\ 9.58 \\ 11.4 \\ 13.38 \\ 15.52 \end{pmatrix} \cdot \text{cm}^2$$

Weight of steel reinforcements

$$W := A \cdot 7850 \frac{\text{kgf}}{\text{m}^3}$$

$$W = \begin{pmatrix} 0.559 \\ 0.994 \\ 1.554 \\ 2.237 \\ 3.045 \\ 3.978 \\ 5.034 \\ 6.215 \\ 7.52 \\ 8.95 \\ 10.503 \\ 12.182 \end{pmatrix} \cdot \frac{\text{kgf}}{\text{m}}$$

ORIGIN := 1

$$n := 1 \dots 9 \quad \text{Area}^{\langle n \rangle} := A \cdot n$$

Bar #	Diameter (mm)	Area of cross section (cm ²) for the number of bars is equal to									Weight (kgf/m)
		1	2	3	4	5	6	7	8	9	
3	9.5	0.71	1.43	2.14	2.85	3.56	4.28	4.99	5.70	6.41	0.559
4	12.7	1.27	2.53	3.80	5.07	6.33	7.60	8.87	10.13	11.40	0.994
5	15.9	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.83	17.81	1.554
6	19.1	2.85	5.70	8.55	11.40	14.25	17.10	19.95	22.80	25.65	2.237
7	22.2	3.88	7.76	11.64	15.52	19.40	23.28	27.16	31.04	34.92	3.045
8	25.4	5.07	10.13	15.20	20.27	25.34	30.40	35.47	40.54	45.60	3.978
9	28.6	6.41	12.83	19.24	25.65	32.07	38.48	44.89	51.30	57.72	5.034
10	31.8	7.92	15.83	23.75	31.67	39.59	47.50	55.42	63.34	71.26	6.215
11	34.9	9.58	19.16	28.74	38.32	47.90	57.48	67.06	76.64	86.22	7.520
12	38.1	11.40	22.80	34.20	45.60	57.00	68.41	79.81	91.21	102.61	8.950
13	41.3	13.38	26.76	40.14	53.52	66.90	80.28	93.66	107.04	120.42	10.503
14	44.5	15.52	31.04	46.55	62.07	77.59	93.11	108.63	124.14	139.66	12.182

Metric Steel Reinforcements

$$D := (6 \ 8 \ 10 \ 12 \ 14 \ 16 \ 18 \ 20 \ 22 \ 25 \ 28 \ 32 \ 36 \ 40)^T \text{ mm}$$

$$A := \frac{\pi \cdot D^2}{4} \quad W := A \cdot 7850 \frac{\text{kgf}}{\text{m}^3}$$

$$n := 1 \dots 9 \quad \text{Area}^{(n)} := A \cdot n$$

Diameter (mm)	Area of cross section (cm ²) for the number of bars is equal to									Weight (kgf/m)
	1	2	3	4	5	6	7	8	9	
6	0.28	0.57	0.85	1.13	1.41	1.70	1.98	2.26	2.54	0.222
8	0.50	1.01	1.51	2.01	2.51	3.02	3.52	4.02	4.52	0.395
10	0.79	1.57	2.36	3.14	3.93	4.71	5.50	6.28	7.07	0.617
12	1.13	2.26	3.39	4.52	5.65	6.79	7.92	9.05	10.18	0.888
14	1.54	3.08	4.62	6.16	7.70	9.24	10.78	12.32	13.85	1.208
16	2.01	4.02	6.03	8.04	10.05	12.06	14.07	16.08	18.10	1.578
18	2.54	5.09	7.63	10.18	12.72	15.27	17.81	20.36	22.90	1.998
20	3.14	6.28	9.42	12.57	15.71	18.85	21.99	25.13	28.27	2.466
22	3.80	7.60	11.40	15.21	19.01	22.81	26.61	30.41	34.21	2.984
25	4.91	9.82	14.73	19.63	24.54	29.45	34.36	39.27	44.18	3.853
28	6.16	12.32	18.47	24.63	30.79	36.95	43.10	49.26	55.42	4.834
32	8.04	16.08	24.13	32.17	40.21	48.25	56.30	64.34	72.38	6.313
36	10.18	20.36	30.54	40.72	50.89	61.07	71.25	81.43	91.61	7.990
40	12.57	25.13	37.70	50.27	62.83	75.40	87.96	100.53	113.10	9.865

4. Safety Provision

$$\text{Required_Strength} \leq \text{Design_Strength}$$

$$U \leq \phi S_n$$

where

U = required strength (factored loads)

ϕS_n = design strength

S_n = nominal strength

ϕ = strength reduction factor

a. Load Combinations

Basic combination $U = 1.2 \cdot D + 1.6 \cdot L$

Roof combination $U = 1.2 \cdot D + 1.6 \cdot L + 1.0 \cdot L_r$

Wind combination $U = 1.2 \cdot D + 1.6 \cdot W + 1.0 \cdot L + 0.5 \cdot L_r$

where

D = dead load

L = live load

L_r = roof live load

W = wind load

b. Strength Reduction Factor

Strength Condition	Strength reduction factor ϕ
Tension-controlled members ($\epsilon_t \geq 0.005$)	$\phi = 0.9$
Compression-controlled ($\epsilon_t \leq 0.002$)	
Spirally reinforced	$\phi = 0.70$
Other	$\phi = 0.65$
Shear and torsion	$\phi = 0.75$

where

ϵ_t = net tensile strain

For spirally reinforced members

$$\phi = \begin{cases} 0.9 & \text{if } \epsilon_t \geq 0.005 \\ 0.70 & \text{if } \epsilon_t \leq 0.002 \\ \frac{1.7 + 200 \cdot \epsilon_t}{3} & \text{otherwise} \end{cases}$$

$$0.7 + \frac{0.9 - 0.7}{0.005 - 0.002} \cdot (\epsilon_t - 0.002)$$

$$0.7 + \frac{200}{3} \cdot (\epsilon_t - 0.002) = \frac{1.7 + 200 \cdot \epsilon_t}{3}$$

For other members

$$\phi = \begin{cases} 0.9 & \text{if } \epsilon_t \geq 0.005 \\ 0.65 & \text{if } \epsilon_t \leq 0.002 \\ \frac{1.45 + 250 \cdot \epsilon_t}{3} & \text{otherwise} \end{cases}$$

5. Loads on Structures (Case of Two-Way Slabs)

Slab dimension

Short side $L_a := 4\text{m}$

Long side $L_b := 6\text{m}$

A. Preliminary Design

Thickness of two-way slab

$$\text{Perimeter} := (L_a + L_b) \cdot 2$$

$$t_{\min} := \frac{\text{Perimeter}}{180} = 111.111 \cdot \text{mm}$$

$$t := \left(\frac{1}{30} \quad \frac{1}{50} \right) \cdot L_a = (133.333 \quad 80) \cdot \text{mm}$$

$$t := 110\text{mm}$$

Section of beam B1

$$L := 6\text{m}$$

$$h := \left(\frac{1}{10} \quad \frac{1}{15} \right) \cdot L = (600 \quad 400) \cdot \text{mm} \quad h := 500\text{mm}$$

$$b := (0.3 \quad 0.6) \cdot h = (150 \quad 300) \cdot \text{mm} \quad b := 250\text{mm}$$

$$\text{For girders} \quad h = \left(\frac{1}{8} \quad \frac{1}{10} \right) \cdot L$$

$$\text{For two-way slab beams} \quad h = \left(\frac{1}{10} \quad \frac{1}{15} \right) \cdot L$$

$$\text{For floor beams} \quad h = \left(\frac{1}{15} \quad \frac{1}{20} \right) \cdot L$$

Section of beam B2

$$L := 4\text{m}$$

$$h := \left(\frac{1}{10} \quad \frac{1}{15} \right) \cdot L = (400 \quad 266.667) \cdot \text{mm} \quad h := 300\text{mm}$$

$$b := (0.3 \quad 0.6) \cdot h = (90 \quad 180) \cdot \text{mm} \quad b := 200\text{mm}$$

B. Loads on Slab

Floor cover	$\text{Cover} := 50\text{mm} \cdot 22 \frac{\text{kN}}{\text{m}^3} = 1.1 \cdot \frac{\text{kN}}{\text{m}^2}$
RC slab	$\text{Slab} := t \cdot 25 \frac{\text{kN}}{\text{m}^3} = 2.75 \cdot \frac{\text{kN}}{\text{m}^2}$
Ceiling	$\text{Ceiling} := 0.40 \frac{\text{kN}}{\text{m}^2}$
M & E	$\text{Mechanical} := 0.20 \frac{\text{kN}}{\text{m}^2}$
Partition	$\text{Partition} := 1.00 \frac{\text{kN}}{\text{m}^2}$
Dead load	$\text{DL} := \text{Cover} + \text{Slab} + \text{Ceiling} + \text{Mechanical} + \text{Partition} = 5.45 \cdot \frac{\text{kN}}{\text{m}^2}$
Live load for Lab	$\text{LL} := 60\text{psf} = 2.873 \cdot \frac{\text{kN}}{\text{m}^2}$
Factored load	$w_u := 1.2 \cdot \text{DL} + 1.6 \cdot \text{LL} = 11.137 \cdot \frac{\text{kN}}{\text{m}^2}$

C. Loads of Wall

$$\text{Void} := 30\text{mm} \cdot 30\text{mm} \cdot 190\text{mm} \cdot 4$$

$$w_{\text{brick.hollow}} := (90\text{mm} \cdot 90\text{mm} \cdot 190\text{mm} - \text{Void}) \cdot 20 \frac{\text{kN}}{\text{m}^3} = 1.744 \cdot \text{kgf}$$

$$w_{\text{brick.solid}} := 45\text{mm} \cdot 90\text{mm} \cdot 190\text{mm} \cdot 20 \frac{\text{kN}}{\text{m}^3} = 1.569 \cdot \text{kgf}$$

$$\rho_{\text{brick.hollow}} := \frac{w_{\text{brick.hollow}}}{90\text{mm} \cdot 90\text{mm} \cdot 190\text{mm}} = 11.111 \cdot \frac{\text{kN}}{\text{m}^3}$$

$$\text{Brick}_{\text{hollow.10}} := \left(120\text{mm} - \text{Void} \cdot \frac{55}{1\text{m}^2} \right) \cdot 20 \frac{\text{kN}}{\text{m}^3} = 1.648 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$\text{Brick}_{\text{hollow.20}} := \left(220\text{mm} - \text{Void} \cdot \frac{110}{1\text{m}^2} \right) \cdot 20 \frac{\text{kN}}{\text{m}^3} = 2.895 \cdot \frac{\text{kN}}{\text{m}^2}$$

D. Loads on Beam B1

Self weight	$SW := 25\text{cm} \cdot (50\text{cm} - 110\text{mm}) \cdot 25 \frac{\text{kN}}{\text{m}^3} = 2.438 \cdot \frac{\text{kN}}{\text{m}}$
Wall	$w_{\text{wall}} := \text{Brick}_{\text{hollow}} \cdot 10 \cdot (3.5\text{m} - 50\text{cm}) = 4.943 \cdot \frac{\text{kN}}{\text{m}}$
Slab	$\alpha := \frac{\frac{4\text{m}}{2}}{6\text{m}} = 0.333 \quad k := 1 - 2 \cdot \alpha^2 + \alpha^3 = 0.815$ $w_{\text{D.slab}} := DL \cdot \frac{4\text{m}}{2} \cdot 2 \cdot k = 17.763 \cdot \frac{\text{kN}}{\text{m}}$ $w_{\text{L.slab}} := LL \cdot \frac{4\text{m}}{2} \cdot 2 \cdot k = 9.363 \cdot \frac{\text{kN}}{\text{m}}$
Dead load	$w_D := SW + w_{\text{wall}} + w_{\text{D.slab}} = 25.143 \cdot \frac{\text{kN}}{\text{m}}$
Live load	$w_L := w_{\text{L.slab}} = 9.363 \cdot \frac{\text{kN}}{\text{m}}$
Factored load	$w_u := 1.2 \cdot w_D + 1.6 \cdot w_L = 45.153 \cdot \frac{\text{kN}}{\text{m}}$

E. Loads on Beam B2

Self weight	$SW := 20\text{cm} \cdot (30\text{cm} - 110\text{mm}) \cdot 25 \frac{\text{kN}}{\text{m}^3} = 0.95 \cdot \frac{\text{kN}}{\text{m}}$
Wall	$w_{\text{wall}} := \text{Brick}_{\text{hollow}} \cdot 10 \cdot (3.5\text{m} - 30\text{cm}) = 5.272 \cdot \frac{\text{kN}}{\text{m}}$
Slab	$\alpha := \frac{\frac{4\text{m}}{2}}{4\text{m}} = 0.5 \quad k := 1 - 2 \cdot \alpha^2 + \alpha^3 = 0.625$ $w_{\text{D.slab}} := DL \cdot \frac{4\text{m}}{2} \cdot 2 \cdot k = 13.625 \cdot \frac{\text{kN}}{\text{m}}$ $w_{\text{L.slab}} := LL \cdot \frac{4\text{m}}{2} \cdot 2 \cdot k = 7.182 \cdot \frac{\text{kN}}{\text{m}}$
Dead load	$w_D := SW + w_{\text{wall}} + w_{\text{D.slab}} = 19.847 \cdot \frac{\text{kN}}{\text{m}}$
Live load	$w_L := w_{\text{L.slab}} = 7.182 \cdot \frac{\text{kN}}{\text{m}}$
Factored load	$w_u := 1.2 \cdot w_D + 1.6 \cdot w_L = 35.308 \cdot \frac{\text{kN}}{\text{m}}$

F. Loads on Column

Tributary area	$B := 4\text{m}$	$L := 6\text{m}$
Slab loads	$P_{D,\text{slab}} := DL \cdot B \cdot L = 130.8 \cdot \text{kN}$ $P_{L,\text{slab}} := LL \cdot B \cdot L = 68.948 \cdot \text{kN}$	
Beam loads	$P_{B1} := 25\text{cm} \cdot (50\text{cm} - 110\text{mm}) \cdot 25 \frac{\text{kN}}{\text{m}^3} \cdot L = 14.625 \cdot \text{kN}$ $P_{B2} := 20\text{cm} \cdot (30\text{cm} - 110\text{mm}) \cdot 25 \frac{\text{kN}}{\text{m}^3} \cdot B = 3.8 \cdot \text{kN}$	
Wall loads	$P_{\text{wall},1} := \text{Brick}_{\text{hollow},10} \cdot (3.5\text{m} - 50\text{cm}) \cdot L = 29.657 \cdot \text{kN}$ $P_{\text{wall},2} := \text{Brick}_{\text{hollow},10} \cdot (3.5\text{m} - 30\text{cm}) \cdot B = 21.089 \cdot \text{kN}$	
SW of column	$SW = (5\% \dots 7\%) \cdot P_D$	

Total loads for number of floors $n := 7$

$$P_D := (P_{D,\text{slab}} + P_{B1} + P_{B2} + P_{\text{wall},1} + P_{\text{wall},2}) \cdot 1.05 \cdot n = 1469.787 \cdot \text{kN}$$

$$P_L := P_{L,\text{slab}} \cdot n = 482.633 \cdot \text{kN}$$

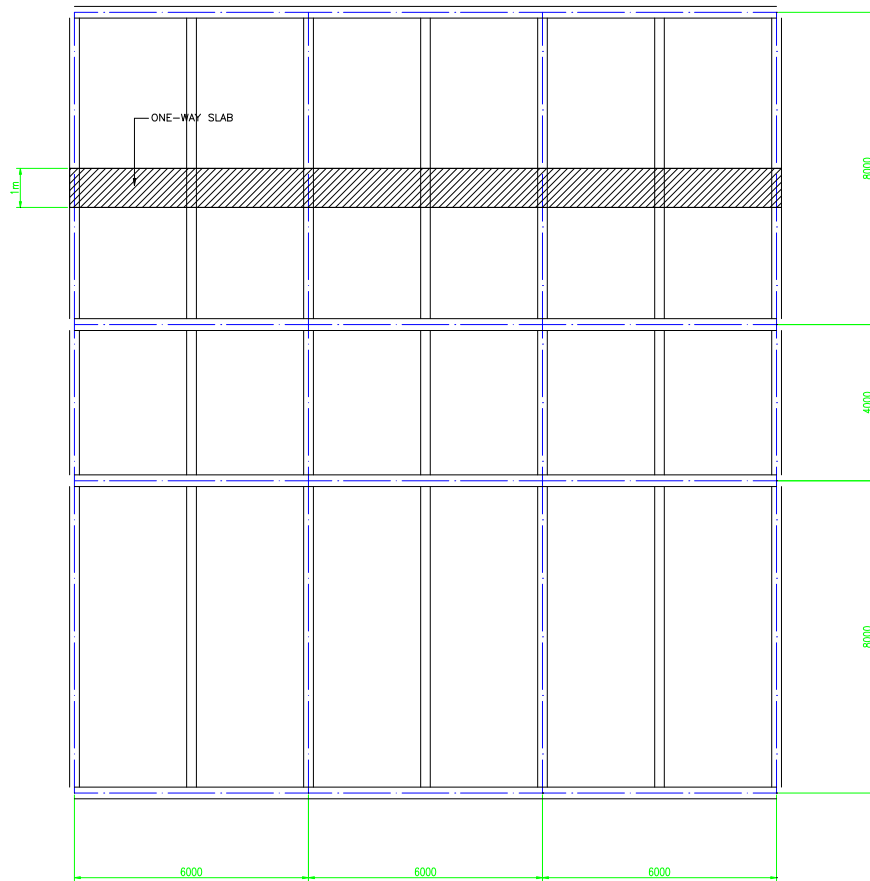
$$P_u := 1.2 \cdot P_D + 1.6 \cdot P_L = 2535.958 \cdot \text{kN} \quad \frac{P_u}{P_D + P_L} = 1.299$$

Control

$$\frac{P_D + P_L}{n \cdot B \cdot L} = 11.622 \cdot \frac{\text{kN}}{\text{m}^2} \quad \left(\text{Ref. } 10 \frac{\text{kN}}{\text{m}^2} \dots 18 \frac{\text{kN}}{\text{m}^2} \right)$$

$$\frac{P_L}{P_D + P_L} = 24.72 \cdot \% \quad (\text{Ref. } 15\% \dots 35\%)$$

06 . Loads on Structures (Case of One-Way Slabs)



A. Preliminary Design

Thickness of one-way slab (both ends continue)

$$L := \frac{6\text{m}}{2} = 3\text{ m}$$

$$t_{\min} := \frac{L}{28} = 107.143 \cdot \text{mm}$$

$$t := \left(\frac{1}{25} \quad \frac{1}{35} \right) \cdot L = (120 \quad 85.714) \cdot \text{mm}$$

$$t := 120\text{mm}$$

Section of floor beam B1

$$L := 8\text{m}$$

$$h := \left(\frac{1}{15} \quad \frac{1}{20} \right) \cdot L = (533.333 \quad 400) \cdot \text{mm} \quad h := 500\text{mm}$$

$$b := (0.3 \quad 0.6) \cdot h = (150 \quad 300) \cdot \text{mm} \quad b := 250\text{mm}$$

Section of girder B2

$$L := 6\text{m}$$

$$h := \left(\frac{1}{8} \quad \frac{1}{10} \right) \cdot L = (750 \quad 600) \cdot \text{mm} \quad h := 600\text{mm}$$

$$b := (0.3 \quad 0.6) \cdot h = (180 \quad 360) \cdot \text{mm} \quad b := 300\text{mm}$$

B. Loads on Slab

$$\text{Cover} := 50\text{mm} \cdot 22 \frac{\text{kN}}{\text{m}^3} = 1.1 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$\text{Slab} := t \cdot 25 \frac{\text{kN}}{\text{m}^3} = 3 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$\text{Ceiling} := 0.40 \frac{\text{kN}}{\text{m}^2}$$

$$\text{Mechanical} := 0.20 \frac{\text{kN}}{\text{m}^2}$$

$$\text{Partition} := 1.00 \frac{\text{kN}}{\text{m}^2}$$

$$\text{DL} := \text{Cover} + \text{Slab} + \text{Ceiling} + \text{Mechanical} + \text{Partition} = 5.7 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$\text{LL} := 60\text{psf} = 2.873 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$w_u := 1.2 \cdot \text{DL} + 1.6 \cdot \text{LL} = 11.437 \cdot \frac{\frac{\text{kN}}{\text{m}}}{1\text{m}}$$

C. Loads on Beam B1

$$\text{Void} := 30\text{mm} \cdot 30\text{mm} \cdot 190\text{mm} \cdot 4$$

$$\text{Brick}_{\text{hollow}.10} := \left(120\text{mm} - \text{Void} \cdot \frac{55}{1\text{m}^2} \right) \cdot 20 \frac{\text{kN}}{\text{m}^3} = 1.648 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$\text{SW} := 25\text{cm} \cdot (50\text{cm} - 120\text{mm}) \cdot 25 \frac{\text{kN}}{\text{m}^3} = 2.375 \cdot \frac{\text{kN}}{\text{m}}$$

$$w_{\text{wall}} := \text{Brick}_{\text{hollow}.10} \cdot (3.5\text{m} - 50\text{cm}) = 4.943 \cdot \frac{\text{kN}}{\text{m}}$$

$$w_{\text{D.slabs}} := \text{DL} \cdot 3\text{m} = 17.1 \cdot \frac{\text{kN}}{\text{m}}$$

$$w_{\text{L.slabs}} := \text{LL} \cdot 3\text{m} = 8.618 \cdot \frac{\text{kN}}{\text{m}}$$

$$w_{\text{D}} := \text{SW} + w_{\text{wall}} + w_{\text{D.slabs}} = 24.418 \cdot \frac{\text{kN}}{\text{m}}$$

$$w_{\text{L}} := w_{\text{L.slabs}} = 8.618 \cdot \frac{\text{kN}}{\text{m}}$$

$$w_{\text{u}} := 1.2 \cdot w_{\text{D}} + 1.6 \cdot w_{\text{L}} = 43.091 \cdot \frac{\text{kN}}{\text{m}}$$

D. Loads on Girder B2

$$\text{SW} := 30\text{cm} \cdot (60\text{cm} - 120\text{mm}) \cdot 25 \frac{\text{kN}}{\text{m}^3} = 3.6 \cdot \frac{\text{kN}}{\text{m}}$$

$$w_{\text{wall}} := \text{Brick}_{\text{hollow}.10} \cdot (3.5\text{m} - 60\text{cm}) = 4.778 \cdot \frac{\text{kN}}{\text{m}}$$

$$P_{\text{B1}} := 25\text{cm} \cdot (50\text{cm} - 120\text{mm}) \cdot 25 \frac{\text{kN}}{\text{m}^3} \cdot \frac{8\text{m} + 4\text{m}}{2} = 14.25 \cdot \text{kN}$$

$$P_{\text{wall}} := \text{Brick}_{\text{hollow}.10} \cdot (3.5\text{m} - 50\text{cm}) \cdot \frac{8\text{m} + 4\text{m}}{2} = 29.657 \cdot \text{kN}$$

$$P_{\text{D.slabs}} := \text{DL} \cdot 3\text{m} \cdot \frac{8\text{m} + 4\text{m}}{2} = 102.6 \cdot \text{kN}$$

$$P_{\text{L.slabs}} := \text{LL} \cdot 3\text{m} \cdot \frac{8\text{m} + 4\text{m}}{2} = 51.711 \cdot \text{kN}$$

Factored loads

$$w_D := SW + w_{\text{wall}} = 8.378 \cdot \frac{\text{kN}}{\text{m}}$$

$$w_L := 0$$

$$w_u := 1.2 \cdot w_D + 1.6 \cdot w_L = 10.054 \cdot \frac{\text{kN}}{\text{m}}$$

$$P_D := P_{B1} + P_{\text{wall}} + P_{D.\text{slab}} = 146.507 \cdot \text{kN}$$

$$P_L := P_{L.\text{slab}} = 51.711 \cdot \text{kN}$$

$$P_u := 1.2 \cdot P_D + 1.6 \cdot P_L = 258.545 \cdot \text{kN}$$

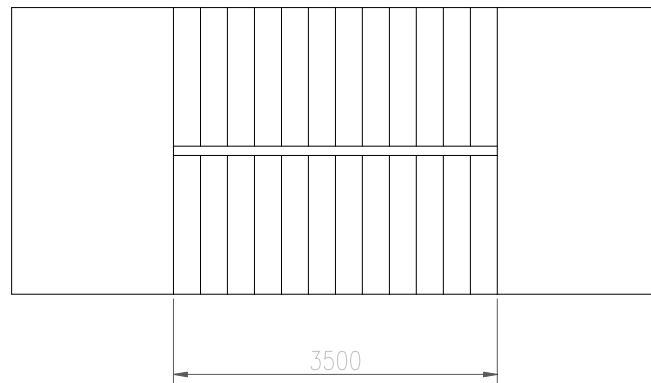
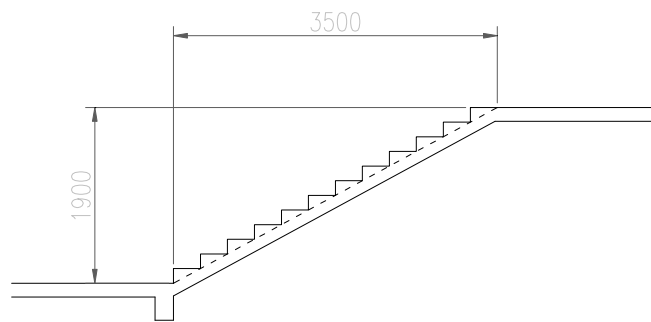
7. Loads on Staircase

Run and rise of step

$$G := \frac{3.5\text{m}}{12} = 291.667 \cdot \text{mm}$$

$$H := \frac{3.8\text{m}}{24} = 158.333 \cdot \text{mm}$$

$$G + 2 \cdot H = 60.833 \cdot \text{cm}$$



Slope angle

$$\alpha := \text{atan}\left(\frac{H}{G}\right) = 28.496 \cdot \text{deg}$$

Loads on Waist Slab

Thickness of waist slab

$$t := 120\text{mm}$$

Step cover

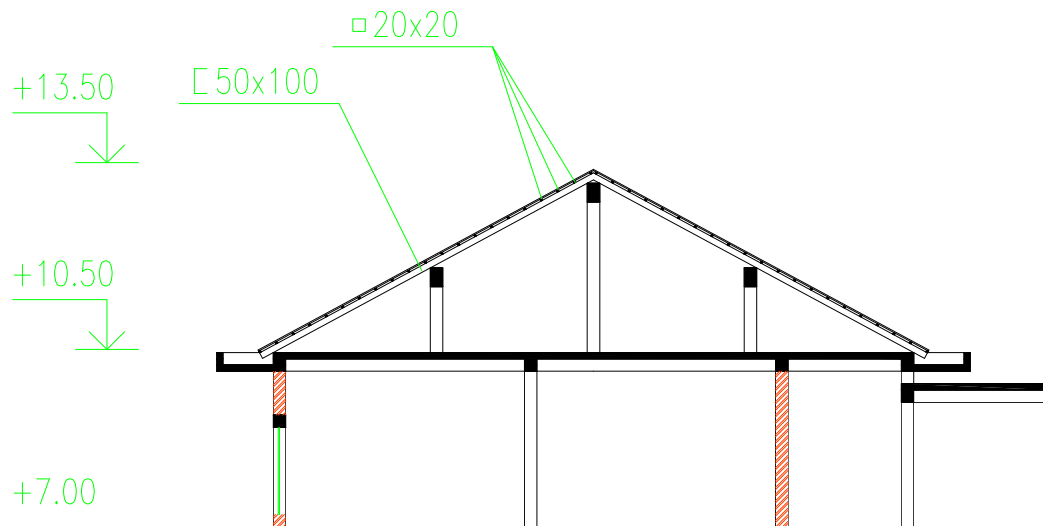
$$\text{Cover} := 50\text{mm} \cdot (H + G) \cdot 22 \frac{\text{kN}}{\text{m}^3} \cdot \frac{1\text{m}}{G \cdot 1\text{m}} = 1.697 \cdot \frac{\text{kN}}{\text{m}^2}$$

SW of step	$\text{Step} := \frac{G \cdot H}{2} \cdot 24 \frac{\text{kN}}{\text{m}^3} \cdot \frac{1\text{m}}{G \cdot 1\text{m}} = 1.9 \cdot \frac{\text{kN}}{\text{m}^2}$
SW of waist slab	$\text{Slab} := t \cdot 25 \frac{\text{kN}}{\text{m}^3} \cdot \frac{1\text{m}^2}{1\text{m}^2 \cdot \cos(\alpha)} = 3.414 \cdot \frac{\text{kN}}{\text{m}^2}$
Renderring	$\text{Renderring} := 0.40 \frac{\text{kN}}{\text{m}^2} \cdot \frac{1\text{m}^2}{1\text{m}^2 \cdot \cos(\alpha)} = 0.455 \cdot \frac{\text{kN}}{\text{m}^2}$
Handrail	$\text{Handrail} := 0.50 \frac{\text{kN}}{\text{m}^2}$
Total dead load	$\text{DL} := \text{Cover} + \text{Step} + \text{Slab} + \text{Renderring} + \text{Handrail}$
	$\text{DL} = 7.966 \cdot \frac{\text{kN}}{\text{m}^2}$
Live load for public staircase	$\text{LL} := 100\text{psf} = 4.788 \cdot \frac{\text{kN}}{\text{m}^2}$
Factored load	$w_u := 1.2 \cdot \text{DL} + 1.6 \cdot \text{LL} = 17.22 \cdot \frac{\text{kN}}{\text{m}^2}$

Loads on Landing Slab

Cover	$:= 50\text{mm} \cdot 22 \frac{\text{kN}}{\text{m}^3} = 1.1 \cdot \frac{\text{kN}}{\text{m}^2}$
Slab	$:= 150\text{mm} \cdot 25 \frac{\text{kN}}{\text{m}^3} = 3.75 \cdot \frac{\text{kN}}{\text{m}^2}$
Renderring	$:= 0.40 \frac{\text{kN}}{\text{m}^2}$
Handrail	$:= 0.50 \frac{\text{kN}}{\text{m}^2}$
DL	$:= \text{Cover} + \text{Slab} + \text{Renderring} + \text{Handrail} = 5.75 \cdot \frac{\text{kN}}{\text{m}^2}$
LL	$:= 100\text{psf} = 4.788 \cdot \frac{\text{kN}}{\text{m}^2}$
w_u	$:= 1.2 \cdot \text{DL} + 1.6 \cdot \text{LL} = 14.561 \cdot \frac{\text{kN}}{\text{m}^2}$

8. Loads on Roof



Slope angle

$$\alpha := \operatorname{atan}\left[\frac{3\text{m}}{\left(\frac{10\text{m}}{2}\right)}\right] = 30.964 \cdot \text{deg}$$

Srokalin tile

$$\text{Tile} := 30\text{mm} \cdot 20 \frac{\text{kN}}{\text{m}^3} = 0.6 \cdot \frac{\text{kN}}{\text{m}^2}$$

Purlins

$$w_{20 \times 20 \times 1.0} := (20\text{mm} \cdot 20\text{mm} - 18\text{mm} \cdot 18\text{mm}) \cdot 7850 \frac{\text{kgf}}{\text{m}^3} = 0.597 \cdot \frac{\text{kgf}}{1\text{m}}$$

$$\text{Purlin} := w_{20 \times 20 \times 1.0} \cdot \frac{1\text{m}}{1\text{m} \cdot 100\text{mm} \cdot \cos(\alpha)} = 0.068 \cdot \frac{\text{kN}}{\text{m}^2}$$

Rafters

$$w_{40 \times 80 \times 1.6} := (40\text{mm} \cdot 80\text{mm} - 36.8\text{mm} \cdot 76.8\text{mm}) \cdot 7850 \frac{\text{kgf}}{\text{m}^3}$$

$$w_{40 \times 80 \times 1.6} = 2.934 \cdot \frac{\text{kgf}}{1\text{m}}$$

$$\text{Rafter} := w_{40 \times 80 \times 1.6} \cdot \frac{1\text{m}}{750\text{mm} \cdot 1\text{m} \cdot \cos(\alpha)} = 0.045 \cdot \frac{\text{kN}}{\text{m}^2}$$

Roof beam	$\text{Beam} := 20\text{cm} \cdot 30\text{cm} \cdot 25 \frac{\text{kN}}{\text{m}^3} \cdot \frac{1\text{m}}{1\text{m} \cdot \frac{10\text{m}}{4}} = 0.6 \cdot \frac{\text{kN}}{\text{m}^2}$
Roof column	$\text{Column} := 20\text{cm} \cdot 20\text{cm} \cdot \frac{3\text{m}}{2} \cdot 25 \frac{\text{kN}}{\text{m}^3} \cdot \frac{1}{\left(\frac{10\text{m}}{4} \cdot 4\text{m}\right)} = 0.15 \cdot \frac{\text{kN}}{\text{m}^2}$
Total dead load	$w_D := \text{Tile} + \text{Purlin} + \text{Rafter} + \text{Beam} + \text{Column} = 1.463 \cdot \frac{\text{kN}}{\text{m}^2}$
Live load	$w_L := 1.00 \frac{\text{kN}}{\text{m}^2}$
Factored load	$w_u := 1.2 \cdot w_D + 1.6 \cdot w_L = 3.356 \cdot \frac{\text{kN}}{\text{m}^2}$

9. ASCE Wind Loads

Basic wind speed	$V := 120 \frac{\text{km}}{\text{hr}}$	$V = 33.333 \frac{\text{m}}{\text{s}}$	$V = 74.565 \text{ mph}$
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Exposure category	Exposure = C
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Importance factor	$I := 1.15$
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Topographic factor	$K_{zt} := 1.0$
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Gust factor	$G := 0.85$
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Wind directionality factor	$K_d := 0.85$
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Static wind pressure	$q_s := 0.613 \frac{\text{N}}{\text{m}^2} \cdot \left(\frac{V}{\frac{\text{m}}{\text{s}}} \right)^2 = 0.681 \frac{\text{kN}}{\text{m}^2}$
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Velocity pressure coefficients

$z_g := 274 \text{ m}$	$\alpha := 9.5$	(For exposure C)
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$K_z(z) := 2.01 \cdot \left(\frac{\max(z, 4.6 \text{ m})}{z_g} \right)^{\frac{2}{\alpha}}$	$K_z(10 \text{ m}) = 1.001$
---	-----------------------------

Velocity wind pressure

$q_z(z) := q_s \cdot K_z(z) \cdot K_{zt} \cdot I \cdot K_d$	$q_z(10 \text{ m}) = 0.667 \frac{\text{kN}}{\text{m}^2}$
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Design wind pressure

$$p_z(z, C_p) := q_z(z) \cdot G \cdot C_p$$

Dimension of building in plan

$$B := 6 \text{ m} \cdot 3 = 18 \text{ m}$$

$$L := 4 \text{ m} \cdot 5 = 20 \text{ m}$$

$$\lambda := \frac{L}{B} = 1.111$$

External pressure coefficients

$$C_{p, \text{windward}} := 0.8$$

$$C_{p,\text{leeward}} := \text{linterp} \left[\begin{pmatrix} 0 \\ 1 \\ 2 \\ 4 \\ 40 \end{pmatrix}, \begin{pmatrix} -0.5 \\ -0.5 \\ -0.3 \\ -0.2 \\ -0.2 \end{pmatrix}, \lambda \right] = -0.478$$

$$C_{p,\text{side}} := -0.7$$

Floor heights

$$H := \begin{pmatrix} 3.5\text{m} \\ 3.5\text{m} \\ 3.5\text{m} \\ 3.5\text{m} \\ 3.5\text{m} \\ 3.5\text{m} \\ 3.5\text{m} \end{pmatrix}$$

$$H := \text{reverse}(H)$$

$$\text{ORIGIN} := 1$$

$$n := \text{rows}(H) = 7$$

$$h := \sum H = 24.5 \text{ m}$$

Wind forces

$$i := 1 \dots n \quad a_i := \sum_{k=1}^i H_k - \frac{H_i}{2}$$

$$a_{n+1} := \sum H$$

$$\text{reverse}(a) = \begin{pmatrix} 24.5 \\ 22.75 \\ 19.25 \\ 15.75 \\ 12.25 \\ 8.75 \\ 5.25 \\ 1.75 \end{pmatrix} \text{ m}$$

$$B_{\text{windward}} := 6\text{m}$$

$$B_{\text{leeward}} := 6\text{m}$$

$$B_{\text{side}} := 4\text{m}$$

$$P_{\text{windward}_i} := \int_{a_i}^{a_{i+1}} p_z(z, C_{p,\text{windward}}) dz \cdot B_{\text{windward}}$$

$$P_{\text{leeward}_i} := p_z(h, C_{p,\text{leeward}}) \cdot (a_{i+1} - a_i) \cdot B_{\text{leeward}}$$

$$P_{\text{side}_i} := p_z(h, C_{p,\text{side}}) \cdot (a_{i+1} - a_i) \cdot B_{\text{side}}$$

$$\text{reverse}\left(\text{augment}\left(P_{\text{windward}}, P_{\text{leeward}}, P_{\text{side}}\right)\right) = \begin{pmatrix} 5.70 & -3.43 & -3.35 \\ 11.13 & -6.86 & -6.71 \\ 10.71 & -6.86 & -6.71 \\ 10.21 & -6.86 & -6.71 \\ 9.61 & -6.86 & -6.71 \\ 8.81 & -6.86 & -6.71 \\ 8.10 & -6.86 & -6.71 \end{pmatrix} \cdot \text{kN}$$

Alternative ways

$$i := 1 \dots n$$

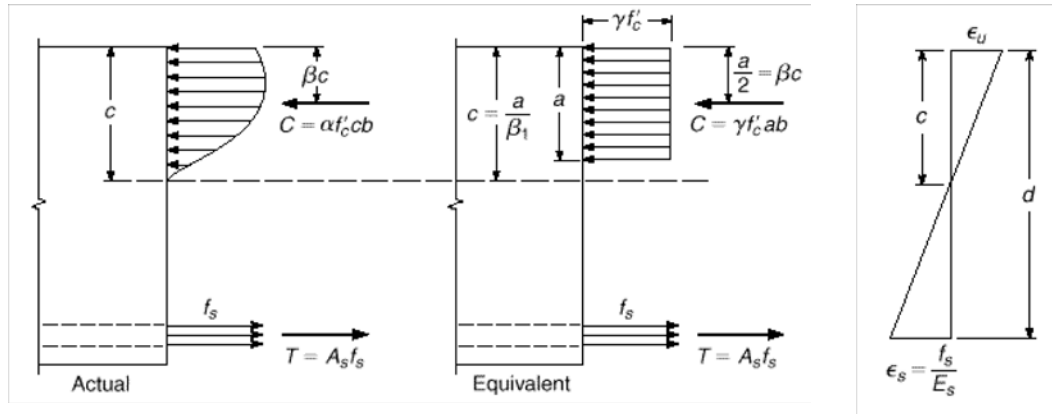
$$b_i := \sum_{k=1}^i H_k$$

$$P_{\text{rectangle}_i} := p_z(b_i, C_{p.\text{windward}}) \cdot (a_{i+1} - a_i) \cdot B_{\text{windward}}$$

$$P_{\text{trapezium}_i} := \frac{p_z(a_i, C_{p.\text{windward}}) + p_z(a_{i+1}, C_{p.\text{windward}})}{2} \cdot (a_{i+1} - a_i) \cdot B_{\text{windward}}$$

$$\text{reverse}\left(\text{augment}\left(P_{\text{windward}}, P_{\text{rectangle}}, P_{\text{trapezium}}\right)\right) = \begin{pmatrix} 5.70 & 5.75 & 5.70 \\ 11.13 & 11.13 & 11.12 \\ 10.71 & 10.71 & 10.70 \\ 10.21 & 10.22 & 10.20 \\ 9.61 & 9.62 & 9.59 \\ 8.81 & 8.83 & 8.78 \\ 8.10 & 8.08 & 8.20 \end{pmatrix} \text{kN}$$

10. Design of Singly Reinforced Beams



A. Concrete Stress Distribution

In actual distribution

Resultant $C = \alpha \cdot f'_c \cdot c \cdot b$

Location $\beta \cdot c$

In equivalent distribution

Location $\beta \cdot c = \frac{a}{2}$

Resultant $C = \alpha \cdot f'_c \cdot c \cdot b = \gamma \cdot f'_c \cdot a \cdot b$

Thus,

$a = 2 \cdot \beta \cdot c = \beta_1 \cdot c$ where $\beta_1 = 2 \cdot \beta$

$$\gamma = \alpha \cdot \frac{c}{a} = \frac{\alpha}{\beta_1}$$

f'_c	4000psi	5000psi	6000psi	7000psi	8000psi
α	0.72	0.68	0.64	0.60	0.56
β	0.425	0.400	0.375	0.350	0.325
$\beta_1 = 2 \cdot \beta$	0.85	0.80	0.75	0.70	0.65
$\gamma = \frac{\alpha}{\beta_1}$	$\frac{0.72}{0.85} = 0.847$	$\frac{0.68}{0.80} = 0.85$	$\frac{0.64}{0.75} = 0.853$	$\frac{0.60}{0.70} = 0.857$	$\frac{0.56}{0.65} = 0.862$

Conclusion: $\gamma = 0.85$

$$\beta_1 = \begin{cases} 0.85 & \text{if } f'_c \leq 4000\text{psi} \\ 0.65 & \text{if } f'_c \geq 8000\text{psi} \\ 0.85 - 0.05 \cdot \frac{f'_c - 4000\text{psi}}{1000\text{psi}} & \text{otherwise} \end{cases}$$

4000psi = 27.6·MPa
8000psi = 55.2·MPa
1000psi = 6.9·MPa

B. Strength Analysis

Equilibrium in forces

$$\sum X = 0$$

$$C = T$$

$$0.85 \cdot f'_c \cdot a \cdot b = A_s \cdot f_s \quad (1)$$

Equilibrium in moments

$$\sum M = 0$$

$$M_n = C \cdot \left(d - \frac{a}{2}\right) = T \cdot \left(d - \frac{a}{2}\right)$$

$$M_n = 0.85 \cdot f'_c \cdot a \cdot b \cdot \left(d - \frac{a}{2}\right) \quad (2.1)$$

$$M_n = A_s \cdot f_s \cdot \left(d - \frac{a}{2}\right) \quad (2.2)$$

Conditions of strain compatibility

$$\frac{\epsilon_s}{\epsilon_u} = \frac{d - c}{c}$$

$$\epsilon_s = \epsilon_u \cdot \frac{d - c}{c} \quad \text{or} \quad \epsilon_t = \epsilon_u \cdot \frac{d_t - c}{c} \quad (3.1)$$

$$c = d \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_s} \quad \text{or} \quad c = d_t \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_t} \quad (3.2)$$

Unknowns = 3 a, A_s, f_s

Equations = 2 $\sum X = 0$ $\sum M = 0$

Additional condition $f_s = f_y$ (From economic criteria)

C. Steel Ratios

$$\rho = \frac{A_s}{b \cdot d} = \frac{A_s \cdot f_y}{b \cdot d \cdot f_y} = \frac{0.85 \cdot f'_c \cdot a \cdot b}{b \cdot d \cdot f_y} = 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{c}{d} = 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{c}{d_t} \cdot \frac{d_t}{d}$$

$$\rho = 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_s} = 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_t} \cdot \frac{d_t}{d}$$

Balanced steel ratio

$$f_c = f'_c \quad f_s = f_y \quad \epsilon_s = \epsilon_y = \frac{f_y}{E_s}$$

$$\rho_b = 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_y} = 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{600 \text{ MPa}}{600 \text{ MPa} + f_y}$$

$$\epsilon_u := 0.003 \quad E_s := 2 \cdot 10^5 \text{ MPa} \quad \epsilon_u \cdot E_s = 600 \cdot \text{MPa}$$

Maximum steel ratio

$$\text{ACI 318-99} \quad \rho_{\max} = 0.75 \cdot \rho_b$$

$$\text{ACI 318-02 and later} \quad \rho_{\max} = 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_t} \quad \text{with } \epsilon_t \geq 0.004$$

$$\text{For } f_y := 390 \text{ MPa} \quad \epsilon_s := \frac{f_y}{E_s} = 0.002$$

$$\text{For } \epsilon_t := 0.004 \quad \frac{\rho_{\max}}{\rho_b} = \frac{\epsilon_u + \epsilon_y}{\epsilon_u + 0.004} = \frac{5}{7} = 0.714$$

$$\text{For } \epsilon_t := 0.005 \quad \frac{\rho_{\max}}{\rho_b} = \frac{\epsilon_u + \epsilon_y}{\epsilon_u + 0.005} = \frac{5}{8} = 0.625$$

Minimum steel ratio

$$\rho_{\min} = \frac{3 \cdot \sqrt{f'_c}}{f_y} \geq \frac{200}{f_y} \quad (\text{in psi})$$

$$\rho_{\min} = \frac{0.249 \cdot \sqrt{f'_c}}{f_y} \geq \frac{1.379}{f_y} \quad (\text{in MPa})$$

D. Determination of Flexural Strength

Given: $b \times d, A_s, f'_c, f_y$

Find: ϕM_n

Step 1. Checking for steel ratio

$$\rho = \frac{A_s}{b \cdot d}$$

$\rho < \rho_{\min}$: Steel reinforcement is not enough

$\rho_{\min} \leq \rho \leq \rho_{\max}$: the beam is singly reinforced

$\rho > \rho_{\max}$: the beam is doubly reinforced

$$\rho = \rho_{\max} \quad A_s = \rho \cdot b \cdot d$$

Step 2. Calculation of flexural strength

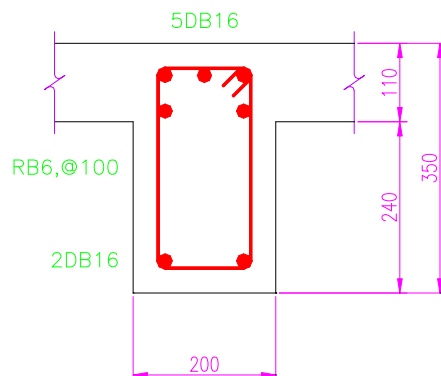
$$a = \frac{A_s \cdot f_y}{0.85 \cdot f'_c \cdot b} \quad c = \frac{a}{\beta_1}$$

$$M_n = A_s \cdot f_y \cdot \left(d - \frac{a}{2} \right)$$

$$\epsilon_t = \epsilon_u \cdot \frac{d_t - c}{c} \quad \phi = \phi(\epsilon_t)$$

The design flexural strength is $\phi \cdot M_n$

Example 10.1



Concrete dimension

$$b := 200\text{mm}$$

$$h := 350\text{mm}$$

Steel reinforcements

$$A_s := 5 \cdot \frac{\pi \cdot (16\text{mm})^2}{4} = 10.053 \cdot \text{cm}^2$$

$$d := h - \left(30\text{mm} + 6\text{mm} + 16\text{mm} + \frac{40\text{mm}}{2} \right) = 278 \cdot \text{mm}$$

$$d_t := h - \left(30\text{mm} + 6\text{mm} + \frac{16\text{mm}}{2} \right) = 306 \cdot \text{mm}$$

Materials

$$f'_c := 25\text{MPa}$$

$$f_y := 390\text{MPa}$$

Solution

Checking for steel ratios

$$\beta_1 := 0.65 \max \left[\left(0.85 - 0.05 \cdot \frac{f'_c - 27.6\text{MPa}}{6.9\text{MPa}} \right) \min 0.85 \right] = 0.85$$

$$\epsilon_u := 0.003$$

$$\rho_{\max} := 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.02$$

$$\rho_{\min} := \max \left(\frac{0.249\text{MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}}}{f_y}, \frac{1.379\text{MPa}}{f_y} \right) = 0.00354$$

$$\rho := \frac{A_s}{b \cdot d} = 0.018$$

$$\text{Steel_Reinforcement} := \begin{cases} \text{"is Enough"} & \text{if } \rho \geq \rho_{\min} \\ \text{"is not Enough"} & \text{otherwise} \end{cases}$$

Steel_Reinforcement = "is Enough"

$$A_s := \min(\rho, \rho_{\max}) \cdot b \cdot d = 10.053 \cdot \text{cm}^2$$

Calculation of flexural strength

$$a := \frac{A_s \cdot f_y}{0.85 \cdot f'_c \cdot b} = 92.252 \cdot \text{mm}$$

$$c := \frac{a}{\beta_1} = 108.532 \cdot \text{mm}$$

$$M_n := A_s \cdot f_y \cdot \left(d - \frac{a}{2} \right) = 90.911 \cdot \text{kN} \cdot \text{m}$$

$$\varepsilon_t := \varepsilon_u \cdot \frac{d_t - c}{c} = 0.00546$$

$$\phi := 0.65 \max \left[\left(\frac{1.45 + 250 \cdot \varepsilon_t}{3} \right) \min 0.9 \right] = 0.9$$

The design flexural strength is $\phi \cdot M_n = 81.82 \cdot \text{kN} \cdot \text{m}$

E. Determination of Steel Area

Given: $M_u, b \times d, f'_c, f_y$

Find: A_s

Relative depth of compression concrete

$$w = \frac{a}{d} = \frac{0.85 \cdot f'_c \cdot a \cdot b}{0.85 \cdot f'_c \cdot b \cdot d} = \frac{A_s \cdot f_y}{0.85 \cdot f'_c \cdot b \cdot d} = \frac{\rho \cdot f_y}{0.85 \cdot f'_c} < 1$$

Flexural resistance factor

$$R = \frac{M_n}{b \cdot d^2} = \frac{A_s \cdot f_y \cdot \left(d - \frac{a}{2} \right)}{b \cdot d^2} = \frac{A_s}{b \cdot d} \cdot f_y \cdot \frac{d - \frac{a}{2}}{d} = \rho \cdot f_y \cdot \left(1 - \frac{1}{2} \cdot w \right)$$

$$R = \rho \cdot f_y \cdot \left(1 - \frac{\rho \cdot f_y}{1.7 \cdot f'_c} \right) = 0.85 \cdot f'_c \cdot w \cdot \left(1 - \frac{1}{2} \cdot w \right)$$

Quadratic equation relative w

$$\frac{R}{0.85 \cdot f'_c} = w \cdot \left(1 - \frac{1}{2} \cdot w \right)$$

$$w^2 - 2 \cdot w + 2 \cdot \frac{R}{0.85 \cdot f'_c} = 0$$

$$w_1 = 1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} < 1$$

$$w_2 = 1 + \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} > 1$$

$$w = 1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}}$$

$$\rho = 0.85 \cdot \frac{f'_c}{f_y} \cdot w = 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right)$$

Step 1. Assume $\phi = 0.9$

$$M_n = \frac{M_u}{\phi}$$

Step 2. Calculation of steel area

$$R = \frac{M_n}{b \cdot d^2}$$

$$\rho = 0.85 \cdot \frac{f_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f_c}} \right)$$

$\rho > \rho_{\max}$: the beam is doubly reinforced
(concrete is not enough)

$\rho \leq \rho_{\max}$: the beam is singly reinforced

$$A_s = \max(\rho, \rho_{\min}) \cdot b \cdot d \quad (\text{this is a required steel area})$$

Step 3. Checking for flexural strength

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b} \quad (A_s \text{ is a provided steel area})$$

$$M_n = A_s \cdot f_y \cdot \left(d - \frac{a}{2} \right)$$

$$c = \frac{a}{\beta_1} \quad \varepsilon_t = \varepsilon_u \cdot \frac{d_t - c}{c} \quad \phi = \phi(\varepsilon_t)$$

$$FS = \frac{M_u}{\phi \cdot M_n} \quad (\text{usage percentage})$$

$FS \leq 1$: the beam is safe

$FS > 1$: the beam is not safe

Example 10.2

Required strength $M_u := 153 \text{ kN}\cdot\text{m}$

Concrete section $b := 200 \text{ mm}$ $h := 500 \text{ mm}$

$$d := h - \left(30 \text{ mm} + 8 \text{ mm} + 18 \text{ mm} + \frac{40 \text{ mm}}{2} \right) = 424 \text{ mm}$$

$$d_t := h - \left(30\text{mm} + 8\text{mm} + \frac{18\text{mm}}{2} \right) = 453\cdot\text{mm}$$

Materials

$$f_c := 25\text{MPa}$$

$$f_y := 390\text{MPa}$$

Solution

Steel ratios

$$\beta_1 := 0.65 \max \left[\left(0.85 - 0.05 \cdot \frac{f_c - 27.6\text{MPa}}{6.9\text{MPa}} \right) \min 0.85 \right] = 0.85$$

$$\epsilon_u := 0.003$$

$$\rho_{\max} := 0.85 \cdot \beta_1 \cdot \frac{f_c}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.02$$

$$\rho_{\min} := \max \left(\frac{0.249\text{MPa} \cdot \sqrt{\frac{f_c}{\text{MPa}}}}{f_y}, \frac{1.379\text{MPa}}{f_y} \right) = 0.00354$$

Assume

$$\phi := 0.9$$

$$M_n := \frac{M_u}{\phi} = 170\cdot\text{kN}\cdot\text{m}$$

Steel area

$$R := \frac{M_n}{b \cdot d^2} = 4.728\cdot\text{MPa}$$

$$\rho := 0.85 \cdot \frac{f_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f_c}} \right) = 0.014$$

$$\rho_{\min} \leq \rho \leq \rho_{\max} = 1$$

$$A_s := \rho \cdot b \cdot d = 11.783\cdot\text{cm}^2$$

$$A_s := 6 \cdot \frac{\pi \cdot (16\text{mm})^2}{4} = 12.064\cdot\text{cm}^2$$

Checking for flexural strength

$$a := \frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b} = 110.702\cdot\text{mm}$$

$$c := \frac{a}{\beta_1} = 130.238\cdot\text{mm}$$

$$M_n := A_s \cdot f_y \cdot \left(d - \frac{a}{2} \right) = 173.444\cdot\text{kN}\cdot\text{m}$$

$$\varepsilon_t := \varepsilon_u \cdot \frac{d_t - c}{c} = 0.00743$$

$$\phi := 0.65 \max \left[\left(\frac{1.45 + 250 \cdot \varepsilon_t}{3} \right) \min 0.9 \right] = 0.9$$

$$FS := \frac{M_u}{\phi \cdot M_n} = 0.98$$

$$\text{The_beam} := \begin{cases} \text{"is safe"} & \text{if } FS \leq 1 \\ \text{"is not safe"} & \text{otherwise} \end{cases}$$

The_beam = "is safe"

F. Determination of Concrete Dimension and Steel Area

Given: M_u, f'_c, f_y

Find: $b \times d, A_s$

Step 1. Determination of concrete dimension

Assume $\varepsilon_t \geq 0.004$ (Usually $\varepsilon_t \geq 0.005$)

$$\rho = 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t}$$

$$R = \rho \cdot f_y \cdot \left(1 - \frac{\rho \cdot f_y}{1.7 \cdot f'_c} \right)$$

$$\phi = \phi(\varepsilon_t)$$

$$M_n = \frac{M_u}{\phi}$$

$$bd^2 = \frac{M_n}{R}$$

Option 1: $b = \frac{\frac{M_n}{R}}{d^2}$

Option 2: $d = \sqrt{\frac{\frac{M_n}{R}}{b}}$

Option 3: $k = \frac{b}{d}$ $d = \sqrt[3]{\frac{\frac{M_n}{R}}{k}}$ $b = k \cdot d$

Step 2. Calculation of steel area

$$R = \frac{M_n}{b \cdot d^2}$$

$$\rho = 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right)$$

$$A_s = \max(\rho, \rho_{\min}) \cdot b \cdot d$$

Step 3. Checking for flexural strength

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f'_c \cdot b} \quad c = \frac{a}{\beta_1}$$

$$M_n = A_s \cdot f_y \cdot \left(d - \frac{a}{2} \right)$$

$$\varepsilon_t = \varepsilon_u \cdot \frac{d_t - c}{c} \quad \phi = \phi(\varepsilon_t)$$

$$FS = \frac{M_u}{\phi \cdot M_n}$$

Example 10.3

Required strength

$$M_u := 700 \text{ kN} \cdot \text{m}$$

Materials

$$f'_c := 25 \text{ MPa}$$

$$f_y := 390 \text{ MPa}$$

Solution

Steel ratios

$$\beta_1 := 0.65 \max \left[\left(0.85 - 0.05 \cdot \frac{f'_c - 27.6 \text{ MPa}}{6.9 \text{ MPa}} \right) \min 0.85 \right] = 0.85$$

$$\varepsilon_u := 0.003$$

$$\rho_{\max} := 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + 0.004} = 0.02$$

$$\rho_{\min} := \max \left(\frac{0.249 \text{ MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}}}{f_y}, \frac{1.379 \text{ MPa}}{f_y} \right) = 0.00354$$

Assume

$$\varepsilon_t := 0.007$$

$$\rho := 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t} = 0.014$$

$$R := \rho \cdot f_y \cdot \left(1 - \frac{\rho \cdot f_y}{1.7 \cdot f'_c} \right) = 4.728 \cdot \text{MPa}$$

$$\phi := 0.65 \max \left[\left(\frac{1.45 + 250 \cdot \varepsilon_t}{3} \right) \min 0.9 \right] = 0.9 \quad M_n := \frac{M_u}{\phi} = 777.778 \cdot \text{kN} \cdot \text{m}$$

Concrete dimension

$$k = \frac{b}{d} \quad k := \frac{400}{600}$$

$$\text{Cover} := 30\text{mm} + 10\text{mm} + 25\text{mm} + \frac{40\text{mm}}{2}$$

$$\text{Cover} = 85 \cdot \text{mm}$$

$$d := \sqrt[3]{\frac{M_n}{\frac{R}{k}}} = 627.231 \cdot \text{mm}$$

$$b := k \cdot d = 418.154 \cdot \text{mm}$$

$$h := \text{Round}(d + \text{Cover}, 50\text{mm}) = 700 \cdot \text{mm} \quad b := \text{Round}(b, 50\text{mm}) = 400 \cdot \text{mm}$$

$$d := h - \text{Cover} = 615 \cdot \text{mm}$$

$$\begin{pmatrix} b \\ h \end{pmatrix} = \begin{pmatrix} 400 \\ 700 \end{pmatrix} \cdot \text{mm}$$

Steel area

$$R := \frac{M_n}{b \cdot d^2} = 5.141 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right) = 0.015$$

$$A_s := \max(\rho, \rho_{\min}) \cdot b \cdot d = 37.741 \cdot \text{cm}^2$$

$$A_s := 8 \cdot \frac{\pi \cdot (25\text{mm})^2}{4} = 39.27 \cdot \text{cm}^2$$

$$d_t := h - \left(30\text{mm} + 10\text{mm} + \frac{25\text{mm}}{2} \right)$$

$$d_t = 647.5 \cdot \text{mm}$$

Checking for flexural strength

$$a := \frac{A_s \cdot f_y}{0.85 \cdot f'_c \cdot b} = 180.18 \cdot \text{mm}$$

$$c := \frac{a}{\beta_1} = 211.976 \cdot \text{mm}$$

$$M_n := A_s \cdot f_y \cdot \left(d - \frac{a}{2} \right) = 803.914 \cdot \text{kN} \cdot \text{m}$$

$$\varepsilon_t := \varepsilon_u \cdot \frac{d_t - c}{c} = 0.00616$$

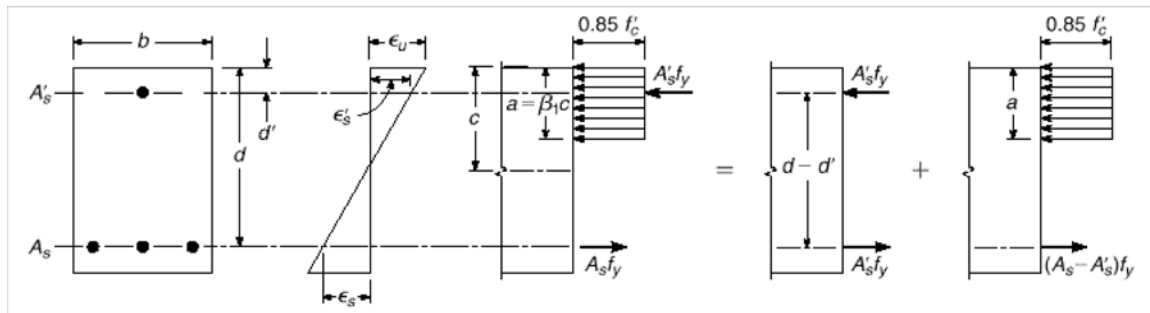
$$\phi := 0.65 \max \left[\left(\frac{1.45 + 250 \cdot \varepsilon_t}{3} \right) \min 0.9 \right] = 0.9$$

$$\text{FS} := \frac{M_u}{\phi \cdot M_n} = 96.749 \cdot \%$$

11. Design of Doubly Reinforced Beams

- $\rho \leq \rho_{\max}$: the beam is singly reinforced
(with tensile reinforcements only)
- $\rho > \rho_{\max}$: the beam is doubly reinforced
(with tensile and compression reinforcements)

A. Strength Analysis



Equilibrium in forces

$$\sum X = 0$$

$$T = C + C_s \quad (1)$$

$$T = A_s \cdot f_s = A_s \cdot f_y$$

$$C = 0.85 \cdot f'_c \cdot a \cdot b$$

$$C_s = A'_s \cdot f'_s$$

Equilibrium in moments

$$\sum M = 0$$

$$M_n = M_{n1} + M_{n2} \quad (2)$$

$$M_{n1} = T \cdot (d - d') = A_s \cdot f_y \cdot (d - d')$$

$$M_{n2} = C \cdot \left(d - \frac{a}{2} \right) = 0.85 \cdot f'_c \cdot a \cdot b \cdot \left(d - \frac{a}{2} \right)$$

$$M_{n2} = (T - C_s) \cdot \left(d - \frac{a}{2} \right) = (A_s \cdot f_y - A'_s \cdot f'_s) \cdot \left(d - \frac{a}{2} \right)$$

Conditions of strain compatibility

$$\frac{\epsilon_s}{\epsilon_u} = \frac{d - c}{c} \quad (3.1)$$

$$\begin{aligned} \epsilon_s &= \epsilon_u \cdot \frac{d - c}{c} & \text{or} & \quad \epsilon_t = \epsilon_u \cdot \frac{d_t - c}{c} \\ c &= d \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_s} & \text{or} & \quad c = d_t \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_t} \end{aligned}$$

$$\frac{\epsilon'_s}{\epsilon_u} = \frac{c - d'}{c} \quad (3.2)$$

$$\begin{aligned} \epsilon'_s &= \epsilon_u \cdot \frac{c - d'}{c} \\ c &= d' \cdot \frac{\epsilon_u}{\epsilon_u - \epsilon'_s} \end{aligned}$$

B. Steel Ratios

Compression steel ratio

$$\rho' = \frac{A'_s}{b \cdot d}$$

Tensile steel ratio

$$\rho = \frac{A_s}{b \cdot d} = \frac{A_s \cdot f_y}{b \cdot d \cdot f_y} = \frac{0.85 \cdot f'_c \cdot a \cdot b + A'_s \cdot f'_s}{b \cdot d \cdot f_y} = 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{c}{d} + \rho' \cdot \frac{f'_s}{f_y}$$

Maximum tensile steel ratio

$$\rho_{t,max} = \rho_{max} + \rho' \cdot \frac{f'_s}{f_y}$$

$\rho \leq \rho_{t,max}$: concrete is enough

$\rho > \rho_{t,max}$: concrete is not enough

Minimum tensile steel ratio

$$\begin{aligned} \rho &= 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{c}{d} + \rho' \cdot \frac{f'_s}{f_y} = 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u - \epsilon'_s} \cdot \frac{d'}{d} + \rho' \cdot \frac{f'_s}{f_y} \\ f'_s &= f_y & \epsilon'_s &= \epsilon_y = \frac{f_y}{E_s} \end{aligned}$$

$$\rho_{cy} = 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u - \epsilon_y} \cdot \frac{d'}{d} + \rho'$$

$\rho \geq \rho_{cy}$: compression steel will yield $f'_s = f_y$

$\rho < \rho_{cy}$: compression steel will not yield $f'_s < f_y$

C. Determination of Flexural Strength

Given: $b \times d, d_t, d', A_s, A'_s, f'_c, f_y$

Find: ϕM_n

Step 1. Checking for singly reinforced beam

$$\rho = \frac{A_s}{b \cdot d}$$

$\rho \leq \rho_{max}$: the beam is singly reinforced

$\rho > \rho_{max}$: the beam is doubly reinforced

$$\rho' = \frac{A'_s}{b \cdot d} \quad \rho_{t,max} = \rho_{max} + \rho'$$

$\rho \leq \rho_{t,max}$: concrete is enough

$\rho > \rho_{t,max}$: concrete is not enough

$$\rho = \rho_{t,max} \quad A_s = \rho \cdot b \cdot d$$

Step 2. Determination of compression parameters

2.1. Assume

$$f'_s = f_y$$

2.2. Calculate

$$a = \frac{A_s \cdot f_y - A'_s \cdot f'_s}{0.85 \cdot f'_c \cdot b}$$

$$c = \frac{a}{\beta_1}$$

$$\epsilon'_s = \epsilon_u \cdot \frac{c - d'}{c}$$

$$f'_{s, revised} = E_s \cdot \epsilon'_s \leq f_y$$

If $f'_{s, revised} \neq f'_s$ then $f'_s = f'_{s, revised}$

Goto 2.2

Direct calculation

Case $f'_s = f_y$

$$a = \frac{A_s \cdot f_y - A'_s \cdot f_y}{0.85 \cdot f'_c \cdot b}$$

Case $f'_s < f_y$

$$A_s \cdot f_y = 0.85 \cdot f'_c \cdot a \cdot b + A'_s \cdot f'_s = 0.85 \cdot f'_c \cdot a \cdot b + A'_s \cdot E_s \cdot \epsilon_u \cdot \frac{c - d'}{c}$$

$$A_s \cdot f_y = 0.85 \cdot f'_c \cdot a \cdot b + A'_s \cdot E_s \cdot \epsilon_u \cdot \frac{\beta_1 \cdot c - \beta_1 \cdot d'}{\beta_1 \cdot c} = 0.85 \cdot f'_c \cdot a \cdot b + A'_s \cdot f_1 \cdot \frac{a - \beta_1 \cdot d'}{a}$$

where $f_1 = E_s \cdot \epsilon_u = 600 \text{ MPa}$

$$0.85 \cdot f'_c \cdot a^2 \cdot b + (A'_s \cdot f_1 - A_s \cdot f_y) \cdot a - A'_s \cdot f_1 \cdot \beta_1 \cdot d' = 0$$

$$\frac{0.85 \cdot f'_c \cdot a^2 \cdot b + (A'_s \cdot f_1 - A_s \cdot f_y) \cdot a - A'_s \cdot f_1 \cdot \beta_1 \cdot d'}{0.85 \cdot f'_c \cdot d^2 \cdot b} = 0$$

$$\left(\frac{a}{d}\right)^2 + \frac{\rho' \cdot f_1 - \rho \cdot f_y}{0.85 \cdot f'_c} \cdot \frac{a}{d} - \frac{\rho' \cdot f_1 \cdot \beta_1}{0.85 \cdot f'_c} \cdot \frac{d'}{d} = 0$$

$$w^2 + 2 \cdot p \cdot w - q = 0 \quad p = \frac{1}{2} \cdot \frac{\rho' \cdot f_1 - \rho \cdot f_y}{0.85 \cdot f'_c} \quad q = \frac{\rho' \cdot f_1 \cdot \beta_1}{0.85 \cdot f'_c} \cdot \frac{d'}{d}$$

$$w_1 = -p + \sqrt{p^2 + q} > 0 \quad w_2 = -p - \sqrt{p^2 + q} < 0$$

$$a = d \cdot (-p + \sqrt{p^2 + q}) \quad c = \frac{a}{\beta_1}$$

$$\epsilon'_s = \epsilon_u \cdot \frac{c - d'}{c} \quad f'_s = E_s \cdot \epsilon'_s \leq f_y$$

Step 3. Calculation of flexural strength

$$M_{n1} = A'_s \cdot f'_s \cdot (d - d')$$

$$M_{n2} = (A_s \cdot f_y - A'_s \cdot f'_s) \cdot \left(d - \frac{a}{2}\right)$$

$$\epsilon_t = \epsilon_u \cdot \frac{d_t - c}{c} \quad \phi = \phi(\epsilon_t)$$

$$\phi M_n = \phi \cdot (M_{n1} + M_{n2})$$

Example 11.1

Concrete dimension	$b := 300\text{mm}$	$h := 550\text{mm}$
Steel reinforcements	$A_s := 8 \cdot \frac{\pi \cdot (20\text{mm})^2}{4} = 25.133 \cdot \text{cm}^2$ $d := h - \left(30\text{mm} + 10\text{mm} + 16\text{mm} + 20\text{mm} + \frac{40\text{mm}}{2} \right)$ $d = 454 \cdot \text{mm}$ $d_t := h - \left(30\text{mm} + 10\text{mm} + 16\text{mm} + \frac{20\text{mm}}{2} \right) = 484 \cdot \text{mm}$ $A'_s := 4 \cdot \frac{\pi \cdot (20\text{mm})^2}{4} = 12.566 \cdot \text{cm}^2$ $d' := 30\text{mm} + 10\text{mm} + \frac{20\text{mm}}{2} = 50 \cdot \text{mm}$	
Materials	$f_c := 25\text{MPa}$	$f_y := 390\text{MPa}$

Solution

Steel ratios

$$\beta_1 := 0.65 \max \left[\left(0.85 - 0.05 \cdot \frac{f_c - 27.6\text{MPa}}{6.9\text{MPa}} \right) \min 0.85 \right] = 0.85$$

$$\epsilon_u := 0.003$$

$$\rho_{\max} := 0.85 \cdot \beta_1 \cdot \frac{f_c}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + 0.005} = 0.0174$$

$$\rho_{\min} := \max \left(\frac{0.249\text{MPa} \cdot \sqrt{\frac{f_c}{\text{MPa}}}}{f_y}, \frac{1.379\text{MPa}}{f_y} \right) = 0.00354$$

Checking for singly reinforced beam

$$\rho := \frac{A_s}{b \cdot d} = 0.0185$$

$$\text{The_beam} := \begin{cases} \text{"is singly reinforced"} & \text{if } \rho \leq \rho_{\max} \\ \text{"is doubly reinforced"} & \text{otherwise} \end{cases}$$

The_beam = "is doubly reinforced"

$$\rho' := \frac{A'_s}{b \cdot d} = 9.226 \times 10^{-3} \quad \rho_{t,\max} := \rho_{\max} + \rho' = 0.027$$

$$\text{Concrete} := \begin{cases} \text{"is enough"} & \text{if } \rho \leq \rho_{t,\max} \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

$$\text{Concrete} = \text{"is enough"}$$

$$\rho := \min(\rho, \rho_{t,\max}) = 0.0185$$

$$A_s := \rho \cdot b \cdot d = 25.133 \cdot \text{cm}^2$$

Determination of compression parameters

$$E_s := 2 \cdot 10^5 \text{ MPa}$$

$$f_1 := E_s \cdot \varepsilon_u = 600 \cdot \text{MPa}$$

$$\varepsilon_y := \frac{f_y}{E_s} = 1.95 \times 10^{-3}$$

$$\rho_{cy} := 0.85 \cdot \beta_1 \cdot \frac{f_c}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u - \varepsilon_y} \cdot \frac{d'}{d} + \rho' = 0.024$$

$$\text{Compression_steel} := \begin{cases} \text{"will yield"} & \text{if } \rho \geq \rho_{cy} \\ \text{"will not yield"} & \text{otherwise} \end{cases}$$

$$\text{Compression_steel} = \text{"will not yield"}$$

$$a := \begin{cases} \frac{A_s \cdot f_y - A'_s \cdot f_y}{0.85 \cdot f_c \cdot b} & \text{if } \text{Compression_steel} = \text{"will yield"} \\ \text{otherwise} \\ \begin{cases} p \leftarrow \frac{1}{2} \cdot \frac{\rho' \cdot f_1 - \rho \cdot f_y}{0.85 \cdot f_c} \\ q \leftarrow \frac{\rho' \cdot f_1 \cdot \beta_1}{0.85 \cdot f_c} \cdot \frac{d'}{d} \\ d \cdot (-p + \sqrt{p^2 + q}) \end{cases} \end{cases}$$

$$a = 90.825 \cdot \text{mm}$$

$$c := \frac{a}{\beta_1} = 106.853 \cdot \text{mm}$$

$$f'_s := \begin{cases} f_y & \text{if } \text{Compression_steel} = \text{"will yield"} \\ \text{otherwise} \\ \begin{cases} \varepsilon'_s \leftarrow \varepsilon_u \cdot \frac{c - d'}{c} \\ \min(E_s \cdot \varepsilon'_s, f_y) \end{cases} \end{cases}$$

$$f'_s = 319.24 \cdot \text{MPa}$$

Flexural strength

$$M_{n1} := A'_s \cdot f'_s \cdot (d - d') = 162.072 \cdot \text{kN} \cdot \text{m}$$

$$M_{n2} := (A_s \cdot f_y - A'_s \cdot f'_s) \cdot \left(d - \frac{a}{2} \right) = 236.576 \cdot \text{kN} \cdot \text{m}$$

$$\varepsilon_t := \varepsilon_u \cdot \frac{d_t - c}{c} = 0.011$$

$$\phi := 0.65 \max \left[\left(\frac{1.45 + 250 \cdot \varepsilon_t}{3} \right) \min 0.9 \right] = 0.9$$

$$\phi M_n := \phi (M_{n1} + M_{n2}) = 358.783 \cdot \text{kN} \cdot \text{m}$$

D. Design of Doubly Reinforced Beam

Given: $M_u, b \times d, d_t, d', f'_c, f_s$

Find: A_s, A'_s

Step 1. Assume ε_t

$$\rho = 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t}$$

$$\phi = \phi(\varepsilon_t)$$

Step 2. Checking for singly reinforced beam

$$A_s = \rho \cdot b \cdot d$$

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f'_c \cdot b}$$

$$M_n = A_s \cdot f_y \cdot \left(d - \frac{a}{2} \right)$$

$M_u \leq \phi M_n$: the beam is singly reinforced

$M_u > \phi M_n$: the beam is doubly reinforced

Step 3. Case of doubly reinforced beam

$$M_{n2} = M_n$$

$$M_{n1} = \frac{M_u}{\phi} - M_{n2}$$

$$c = \frac{a}{\beta_1} \quad f'_s = E_s \cdot \epsilon_u \cdot \frac{c - d'}{c} \leq f_y$$

$$A'_s = \frac{M_{n1}}{f'_s \cdot (d - d')} \quad \rho' = \frac{A'_s}{b \cdot d} \quad \rho_{t,max} = \rho_{max} + \rho'$$

$$A_s = \frac{0.85 \cdot f'_c \cdot a \cdot b + A'_s \cdot f'_s}{f_y} \quad \rho = \frac{A_s}{b \cdot d}$$

$$\rho \leq \rho_{t,max} \quad : \text{concrete is enough}$$

$$\rho > \rho_{t,max} \quad : \text{concrete is not enough}$$

Example 11.2

Required strength	$M_u := 1350 \text{ kN} \cdot \text{m}$
Concrete dimension	$b := 400 \text{ mm} \quad h := 800 \text{ mm}$ $d := h - \left(30 \text{ mm} + 12 \text{ mm} + 25 \text{ mm} + 25 \text{ mm} + \frac{40 \text{ mm}}{2} \right)$ $d = 688 \cdot \text{mm}$ $d_t := h - \left(30 \text{ mm} + 12 \text{ mm} + 25 \text{ mm} + \frac{25 \text{ mm}}{2} \right) = 720.5 \cdot \text{mm}$ $d' := 30 \text{ mm} + 12 \text{ mm} + \frac{25 \text{ mm}}{2} = 54.5 \cdot \text{mm}$
Materials	$f'_c := 25 \text{ MPa} \quad f_y := 390 \text{ MPa}$

Solution

Steel ratios

$$\beta_1 := 0.65 \max \left[\left(0.85 - 0.05 \cdot \frac{f'_c - 27.6 \text{ MPa}}{6.9 \text{ MPa}} \right) \min 0.85 \right] = 0.85$$

$$\epsilon_u := 0.003$$

$$\rho_{max} := 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + 0.005} = 0.0174$$

$$\rho_{min} := \max \left(\frac{0.249 \text{ MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}}}{f_y}, \frac{1.379 \text{ MPa}}{f_y} \right) = 0.00354$$

Assume

$$\epsilon_t := 0.0092$$

$$\rho := 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_t} = 0.011$$

$$\phi := 0.65 \max \left[\left(\frac{1.45 + 250 \cdot \epsilon_t}{3} \right) \min 0.9 \right] = 0.9$$

Checking for singly reinforced beam

$$A_s := \rho \cdot b \cdot d = 31.342 \cdot \text{cm}^2$$

$$a := \frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b} = 143.803 \cdot \text{mm} \quad c := \frac{a}{\beta_1} = 169.18 \cdot \text{mm}$$

$$M_{n2} := A_s \cdot f_y \cdot \left(d - \frac{a}{2} \right) = 753.074 \cdot \text{kN} \cdot \text{m}$$

$$\text{The_beam} := \begin{cases} \text{"is singly reinforced"} & \text{if } M_u \leq \phi \cdot M_{n2} \\ \text{"is doubly reinforced"} & \text{otherwise} \end{cases}$$

The_beam = "is doubly reinforced"

Case of doubly reinforced beam

$$M_{n1} := \frac{M_u}{\phi} - M_{n2} = 746.926 \cdot \text{kN} \cdot \text{m}$$

$$f'_s := \min \left(E_s \cdot \epsilon_u \cdot \frac{c - d'}{c}, f_y \right) = 390 \cdot \text{MPa}$$

$$A'_s := \frac{M_{n1}}{f'_s \cdot (d - d')} = 30.232 \cdot \text{cm}^2$$

$$\rho' := \frac{A'_s}{b \cdot d} = 0.011$$

$$\rho_{t,\max} := \rho_{\max} + \rho' = 0.028$$

$$A_s := \frac{0.85 \cdot f_c \cdot a \cdot b + A'_s \cdot f'_s}{f_y} = 61.574 \cdot \text{cm}^2$$

$$\rho := \frac{A_s}{b \cdot d} = 0.022$$

$$\text{Concrete} := \begin{cases} \text{"is enough"} & \text{if } \rho \leq \rho_{t,\max} \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

Concrete = "is enough"

Compression steel

$$A'_s = 30.232 \cdot \text{cm}^2$$

$$5 \cdot \frac{\pi \cdot (28\text{mm})^2}{4} = 30.788 \cdot \text{cm}^2$$

Tensile steel

$$A_s = 61.574 \cdot \text{cm}^2$$

$$10 \cdot \frac{\pi \cdot (28\text{mm})^2}{4} = 61.575 \cdot \text{cm}^2$$

$$\frac{400\text{mm} - (12\text{mm} + 30\text{mm}) \cdot 2 - 28\text{mm} \cdot 5}{4} = 44 \cdot \text{mm}$$

E. Determination of Tensile Steel Area

Given: $M_u, b \times d, d_t, d', A'_s, f'_c, f_y$

Find: A_s

Step 1. Calculation of compression parameters

1.1. Assume $f'_s = f_y$ $\phi = 0.9$

1.2. Calculate

$$M_n = \frac{M_u}{\phi}$$

$$M_{n1} = A'_s \cdot f'_s \cdot (d - d')$$

$$M_{n2} = M_n - M_{n1}$$

$$R = \frac{M_{n2}}{b \cdot d^2}$$

$$\rho = 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right)$$

$$A_s = \rho \cdot b \cdot d$$

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f'_c \cdot b} \quad c = \frac{a}{\beta_1}$$

$$f'_{s, \text{revised}} = E_s \cdot \epsilon_u \cdot \frac{c - d'}{c} \leq f_y$$

$$\epsilon_t = \epsilon_u \cdot \frac{d_t - c}{c} \quad \phi = \phi(\epsilon_t)$$

$$f'_{s, \text{revised}} \neq f'_s \quad : \text{Goto 1.2}$$

$$M_u > \phi \cdot M_n \quad : \text{Goto 1.2}$$

Step 2. Calculation of tensile steel area

$$A_s = \frac{0.85 \cdot f'_c \cdot a \cdot b + A'_s \cdot f'_s}{f_y} \quad \rho = \frac{A_s}{b \cdot d}$$

$$\rho' = \frac{A'_s}{b \cdot d} \quad \rho_{t, \text{max}} = \rho_{\text{max}} + \rho'$$

$$\rho \leq \rho_{t, \text{max}} \quad : \text{concrete is enough}$$

$$\rho > \rho_{t, \text{max}} \quad : \text{concrete is not enough}$$

Example 11.3

Required strength	$M_u := 1350 \text{ kN}\cdot\text{m}$
Concrete dimension	$b := 400 \text{ mm}$ $h := 800 \text{ mm}$ $d := h - \left(30 \text{ mm} + 12 \text{ mm} + 25 \text{ mm} + 28 \text{ mm} + \frac{40 \text{ mm}}{2} \right)$ $d = 685 \cdot \text{mm}$ $d_t := h - \left(30 \text{ mm} + 12 \text{ mm} + 25 \text{ mm} + \frac{28 \text{ mm}}{2} \right) = 719 \cdot \text{mm}$ $d' := 30 \text{ mm} + 12 \text{ mm} + \frac{28 \text{ mm}}{2} = 56 \cdot \text{mm}$
Compression reinforcements	$A'_s := 5 \cdot \frac{\pi \cdot (28 \text{ mm})^2}{4} = 30.788 \cdot \text{cm}^2$
Materials	$f'_c := 25 \text{ MPa}$ $f_y := 390 \text{ MPa}$

Solution

Steel ratios

$$\beta_1 := 0.65 \max \left[\left(0.85 - 0.05 \cdot \frac{f'_c - 27.6 \text{ MPa}}{6.9 \text{ MPa}} \right) \min 0.85 \right] = 0.85$$

$$\varepsilon_u := 0.003$$

$$\rho_{\max} := 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + 0.005} = 0.0174$$

$$\rho_{\min} := \max \left(\frac{0.249 \text{ MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}}}{f_y}, \frac{1.379 \text{ MPa}}{f_y} \right) = 0.00354$$

Compression parameters

```

Compression( $\epsilon$ ) :=
   $f'_s \leftarrow f_y$ 
   $\phi \leftarrow 0.9$ 
  for  $i \in 0 \dots 99$ 
     $M_n \leftarrow \frac{M_u}{\phi}$ 
     $M_{n1} \leftarrow A'_s \cdot f'_s \cdot (d - d')$ 
     $M_{n2} \leftarrow M_n - M_{n1}$ 
     $R \leftarrow \frac{M_{n2}}{b \cdot d^2}$ 
     $\rho \leftarrow 0.85 \cdot \frac{f'_c}{f_y} \cdot \left( 1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right)$ 
     $A_s \leftarrow \rho \cdot b \cdot d$ 
     $a \leftarrow \frac{A_s \cdot f_y}{0.85 \cdot f'_c \cdot b}$ 
     $c \leftarrow \frac{a}{\beta_1}$ 
     $f'_{s, \text{revised}} \leftarrow \min \left( E_s \cdot \epsilon_u \cdot \frac{c - d'}{c}, f_y \right)$ 
     $Z^{(i)} \leftarrow \begin{pmatrix} \frac{f'_s}{f_y} \\ \phi \\ \frac{a}{d} \\ \frac{f'_{s, \text{revised}}}{f_y} \end{pmatrix}$ 
     $\epsilon_t \leftarrow \epsilon_u \cdot \frac{d_t - c}{c}$ 
     $\phi \leftarrow 0.65 \max \left[ \left( \frac{1.45 + 250 \cdot \epsilon_t}{3} \right) \min 0.9 \right]$ 
    (break) if  $\left( \left| \frac{f'_{s, \text{revised}} - f'_s}{f'_s} \right| \leq \epsilon \right) \wedge (M_u \leq \phi \cdot M_n)$ 
     $f'_s \leftarrow f'_{s, \text{revised}}$ 
  reverse( $Z^T$ )

```

$Z := \text{Compression}(0.000001)$

$$a := Z_{0,2} \cdot d = 142.792 \cdot \text{mm}$$

$$c := \frac{a}{\beta_1} = 167.99 \cdot \text{mm}$$

$$f'_s := Z_{0,0} \cdot f_y = 390 \cdot \text{MPa}$$

Tensile steel area

$$\rho' := \frac{A'_s}{b \cdot d} = 0.011$$

$$\rho_{t,\max} := \rho_{\max} + \rho' = 0.029$$

$$A_s := \frac{0.85 \cdot f'_c \cdot a \cdot b + A'_s \cdot f'_s}{f_y} = 61.909 \cdot \text{cm}^2$$

$$\rho := \frac{A_s}{b \cdot d} = 0.023$$

$$\text{Concrete} := \begin{cases} \text{"is enough"} & \text{if } \rho \leq \rho_{t,\max} \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

Concrete = "is enough"

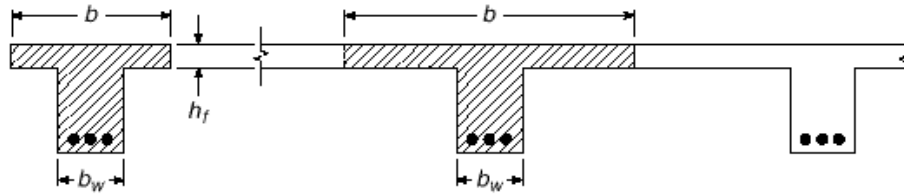
Tensile steel

$$A_s = 61.909 \cdot \text{cm}^2$$

$$10 \cdot \frac{\pi \cdot (28 \text{mm})^2}{4} = 61.575 \cdot \text{cm}^2$$

12. Design of T Beams

12.1. Effective Flange Width



For symmetrical T beam:

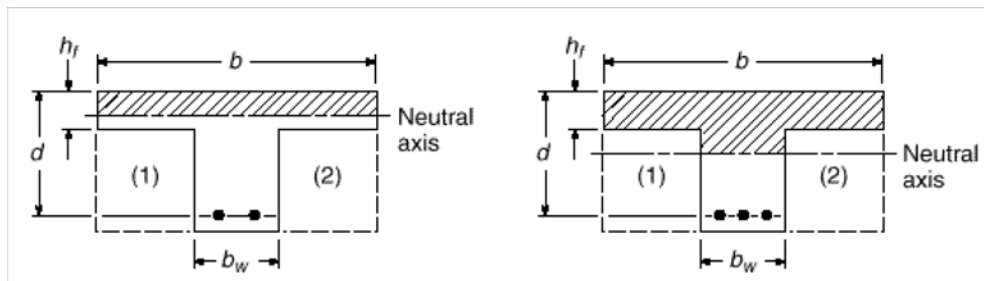
$$b \leq \frac{L}{4} \quad b \leq b_w + 16h_f \quad b \leq s$$

where

L = span length of beam

s = spacing of beam

12.2. Strength Analysis



Design as rectangular section

$$a \leq h_f$$

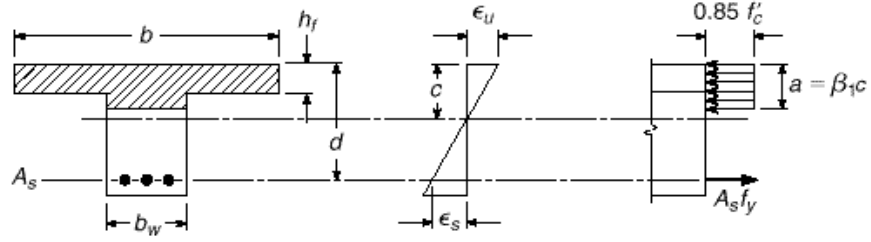
or $M_u \leq \phi M_{nf}$

Design as T section

$$a > h_f$$

or $M_u > \phi M_{nf}$

where $M_{nf} = 0.85 \cdot f'_c \cdot h_f \cdot b \cdot \left(d - \frac{h_f}{2} \right)$



Equilibrium in forces $\sum X = 0$

$$T = C_1 + C_2$$

$$T = A_s \cdot f_s = A_s \cdot f_y$$

$$C_1 = 0.85 \cdot f'_c \cdot h_f \cdot (b - b_w) = A_{sf} \cdot f_y$$

$$C_2 = 0.85 \cdot f'_c \cdot a \cdot b_w = T - C_1 = A_s \cdot f_y - A_{sf} \cdot f_y$$

Equilibrium in moments $\sum M = 0$

$$M_n = M_{n1} + M_{n2}$$

$$M_{n1} = C_1 \cdot \left(d - \frac{h_f}{2} \right) = A_{sf} \cdot f_y \cdot \left(d - \frac{h_f}{2} \right)$$

$$M_{n2} = C_2 \cdot \left(d - \frac{a}{2} \right) = 0.85 \cdot f'_c \cdot a \cdot b_w \cdot \left(d - \frac{a}{2} \right)$$

$$M_{n2} = (T - C_1) \cdot \left(d - \frac{a}{2} \right) = (A_s \cdot f_y - A_{sf} \cdot f_y) \cdot \left(d - \frac{a}{2} \right)$$

Condition of strain compatibility

$$\frac{\epsilon_s}{\epsilon_u} = \frac{d - c}{c} \quad \text{or} \quad \frac{\epsilon_t}{\epsilon_s} = \frac{d_t - c}{c}$$

$$\epsilon_s = \epsilon_u \cdot \frac{d - c}{c} \quad \epsilon_t = \epsilon_u \cdot \frac{d_t - c}{c}$$

$$c = d \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_s} \quad c = d_t \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_t}$$

12.3. Steel Ratios

$$\rho_w = \frac{A_s}{b_w \cdot d} = \frac{A_s \cdot f_y}{b_w \cdot d \cdot f_y} = \frac{0.85 \cdot f'_c \cdot a \cdot b_w + A_{sf} \cdot f_y}{b_w \cdot d \cdot f_y}$$

$$\rho_w = 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{c}{d} + \rho_f \quad \text{where} \quad \rho_f = \frac{A_{sf}}{b_w \cdot d}$$

Maximum steel ratio

$$\rho_{w,max} = \rho_{max} + \rho_f$$

$$\rho_w \leq \rho_{w,max} \quad : \text{concrete is enough}$$

$$\rho_w > \rho_{w,max} \quad : \text{concrete is not enough}$$

12.4. Determination of Moment Capacity

Given: $b_w \times d, b, d_t, h_f, A_s, f'_c, f_y$

Find: ϕM_n

Step 1. Checking for rectangular beam

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f'_c \cdot b}$$

$$a \leq h_f \quad : \text{the beam is rectangular}$$

$$a > h_f \quad : \text{the beam is tee}$$

Step 2. Case of T beam

$$A_{sf} = \frac{0.85 \cdot f'_c \cdot h_f \cdot (b - b_w)}{f_y}$$

$$M_{n1} = A_{sf} \cdot f_y \cdot \left(d - \frac{h_f}{2} \right)$$

$$a = \frac{A_s \cdot f_y - A_{sf} \cdot f_y}{0.85 \cdot f'_c \cdot b_w} \quad c = \frac{a}{\beta_1}$$

$$M_{n2} = (A_s \cdot f_y - A_{sf} \cdot f_y) \cdot \left(d - \frac{a}{2} \right)$$

$$\epsilon_t = \epsilon_u \cdot \frac{d_t - c}{c} \quad \phi = \phi(\epsilon_t)$$

$$\phi M_n = \phi \cdot (M_{n1} + M_{n2})$$

Example 12.1

Moment capacity of a given section. An isolated T beam is composed of a flange 28 in. wide and 6 in. deep cast monolithically with a web of 10 in. width that extends 24 in. below the bottom surface of the flange to produce a beam of 30 in. total depth. Tensile reinforcement consists of six No. 10 (No. 32) bars placed in two horizontal rows. The centroid of the bar group is 26 in. from the top of the beam. It has been determined that the concrete has a strength of 3000 psi and that the yield stress of the steel is 60,000 psi. What is the design moment capacity of the beam?

Concrete dimension	$b := 28\text{in} = 711.2\cdot\text{mm}$	
	$h_f := 6\text{in} = 152.4\cdot\text{mm}$	
	$b_w := 10\text{in} = 254\cdot\text{mm}$	
	$h := 30\text{in} = 762\cdot\text{mm}$	$d := 26\text{in} = 660.4\cdot\text{mm}$
	$d_t := 27.5\text{in} = 698.5\cdot\text{mm}$	
Steel reinforcements	$A_s := 6 \cdot \frac{\pi \cdot \left(\frac{10}{8}\text{in}\right)^2}{4}$	$A_s = 7.363\cdot\text{in}^2$
Materials	$f_c := 3000\text{psi} = 20.684\cdot\text{MPa}$	
	$f_y := 60\text{ksi} = 413.685\cdot\text{MPa}$	

Solution

Steel ratios

$$\beta_1 := 0.65 \max \left[\left(0.85 - 0.05 \cdot \frac{f_c - 4000\text{psi}}{1000\text{psi}} \right) \min 0.85 \right] = 0.85$$

$$\epsilon_u := 0.003$$

$$\rho_{\max} := 0.85 \cdot \beta_1 \cdot \frac{f_c}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + 0.005} = 0.014$$

$$\rho_{\min} := \max \left(\frac{3\text{psi} \cdot \sqrt{\frac{f_c}{\text{psi}}}}{f_y}, \frac{200\text{psi}}{f_y} \right) = 0.00333$$

Checking for rectangular beam

$$a := \frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b} = 157.162 \cdot \text{mm}$$

$$\text{The_beam} := \begin{cases} \text{"is rectangular"} & \text{if } a \leq h_f \\ \text{"is T"} & \text{otherwise} \end{cases}$$

$$\text{The_beam} = \text{"is T"}$$

Case of T beam

$$A_{sf} := \frac{0.85 \cdot f_c \cdot h_f \cdot (b - b_w)}{f_y} = 29.613 \cdot \text{cm}^2$$

$$\rho_f := \frac{A_{sf}}{b_w \cdot d} = 0.018$$

$$\rho_{w,\max} := \rho_{\max} + \rho_f = 0.031$$

$$\rho_w := \frac{A_s}{b_w \cdot d} = 0.028$$

$$\text{Concrete} := \begin{cases} \text{"is enough"} & \text{if } \rho_w \leq \rho_{w,\max} \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

$$\text{Concrete} = \text{"is enough"}$$

$$A_s := \min(\rho_w, \rho_{w,\max}) \cdot b_w \cdot d$$

$$A_s = 7.363 \cdot \text{in}^2$$

$$M_{n1} := A_{sf} \cdot f_y \cdot \left(d - \frac{h_f}{2} \right) = 715.669 \cdot \text{kN} \cdot \text{m}$$

$$a := \frac{A_s \cdot f_y - A_{sf} \cdot f_y}{0.85 \cdot f_c \cdot b_w} = 165.734 \cdot \text{mm}$$

$$c := \frac{a}{\beta_1} = 194.981 \cdot \text{mm}$$

$$M_{n2} := (A_s \cdot f_y - A_{sf} \cdot f_y) \cdot \left(d - \frac{a}{2} \right) = 427.446 \cdot \text{kN} \cdot \text{m}$$

$$\varepsilon_t := \varepsilon_u \cdot \frac{d_t - c}{c} = 0.008$$

$$\phi := 0.65 \max \left[\left(\frac{1.45 + 250 \cdot \varepsilon_t}{3} \right) \min 0.9 \right] = 0.9$$

$$\phi M_n := \phi \cdot (M_{n1} + M_{n2}) = 1028.803 \cdot \text{kN} \cdot \text{m}$$

12.5. Determination of Steel Area

Given: $M_u, b_w \times d, d_t, b, h_f, f_c, f_y$

Find: A_s

Step 1. Checking for rectangular beam

$$M_{nf} = 0.85 \cdot f_c \cdot h_f \cdot b \cdot \left(d - \frac{h_f}{2} \right)$$

$$\phi = 0.9$$

$M_u \leq \phi M_n$: the beam is rectangular

$M_u > \phi M_n$: the beam is tee

Step 2. Case of T beam

$$A_{sf} = \frac{0.85 \cdot f_c \cdot h_f \cdot (b - b_w)}{f_y}$$

$$\rho_f = \frac{A_{sf}}{b_w \cdot d}$$

$$M_{n1} = A_{sf} \cdot f_y \cdot \left(d - \frac{h_f}{2} \right)$$

$$M_{n2} = \frac{M_u}{\phi} - M_{n1}$$

$$R = \frac{M_{n2}}{b_w \cdot d^2}$$

$$\rho = 0.85 \cdot \frac{f_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f_c}} \right)$$

$$A_{s2} = \rho \cdot b_w \cdot d$$

$$a = \frac{A_{s2} \cdot f_y}{0.85 \cdot f_c \cdot b_w}$$

$$A_s = \frac{0.85 \cdot f_c \cdot a \cdot b_w + A_{sf} \cdot f_y}{f_y}$$

$$\rho_w = \frac{A_s}{b_w \cdot d}$$

$$\rho_{w,max} = \rho_{max} + \rho_f$$

$\rho_w \leq \rho_{w,max}$: concrete is enough

$\rho_w > \rho_{w,max}$: concrete is not enough

Example 12.2

Determination of steel area for a given moment. A floor system consists of a 3 in. concrete slab supported by continuous T beams with a 24 ft span, 47 in. on centers. Web dimensions, as determined by negative-moment requirements at the supports, are $b_w = 11$ in. and $d = 20$ in. What tensile steel area is required at midspan to resist a factored moment of 6400 in-kips if $f_y = 60,000$ psi and $f'_c = 3000$ psi?

Concrete dimension	$h_f := 3\text{in} = 76.2\cdot\text{mm}$	
	$L := 24\text{ft} = 7.315\text{ m}$	$s := 47\text{in} = 1.194\cdot\text{m}$
	$b_w := 11\text{in} = 279.4\cdot\text{mm}$	$d := 20\text{in} = 508\cdot\text{mm}$
Required strength	$M_u := 6400\text{in}\cdot\text{kip} = 723.103\cdot\text{kN}\cdot\text{m}$	
Materials	$f'_c := 3000\text{psi} = 20.684\cdot\text{MPa}$	$f_y := 60\text{ksi} = 413.685\cdot\text{MPa}$

Solution

Effective flange width

$$b := \min\left(\frac{L}{4}, b_w + 16\cdot h_f, s\right) \quad b = 1193.8\cdot\text{mm}$$

Steel ratios

$$\beta_1 := 0.65 \max\left[\left(0.85 - 0.05\cdot\frac{f'_c - 4000\text{psi}}{1000\text{psi}}\right) \min 0.85\right] = 0.85$$

$$\epsilon_u := 0.003$$

$$\rho_{\max} := 0.85\cdot\beta_1\cdot\frac{f'_c}{f_y}\cdot\frac{\epsilon_u}{\epsilon_u + 0.005} = 0.014$$

$$\rho_{\min} := \max\left(\frac{3\text{psi}\cdot\sqrt{\frac{f'_c}{\text{psi}}}}{f_y}, \frac{200\text{psi}}{f_y}\right) = 0.00333$$

Checking for rectangular beam

$$\phi := 0.9$$

$$M_{nf} := 0.85\cdot f'_c\cdot h_f\cdot b\cdot\left(d - \frac{h_f}{2}\right) = 751.538\cdot\text{kN}\cdot\text{m}$$

$$\text{The_beam} := \begin{cases} \text{"is rectangular"} & \text{if } M_u \leq \phi\cdot M_{nf} \\ \text{"is tee"} & \text{otherwise} \end{cases}$$

The_beam = "is tee"

Case of T beam

$$A_{sf} := \frac{0.85 \cdot f'_c \cdot h_f \cdot (b - b_w)}{f_y} = 29.613 \cdot \text{cm}^2$$

$$\rho_f := \frac{A_{sf}}{b_w \cdot d} = 0.021$$

$$M_{n1} := A_{sf} \cdot f_y \cdot \left(d - \frac{h_f}{2} \right) = 575.646 \cdot \text{kN} \cdot \text{m}$$

$$M_{n2} := \frac{M_u}{\phi} - M_{n1} = 227.801 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_{n2}}{b_w \cdot d^2} = 3.159 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right) = 8.484 \times 10^{-3}$$

$$A_{s2} := \rho \cdot b_w \cdot d = 12.042 \cdot \text{cm}^2$$

$$a := \frac{A_{s2} \cdot f_y}{0.85 \cdot f'_c \cdot b_w} = 101.408 \cdot \text{mm}$$

$$c := \frac{a}{\beta_1} = 119.304 \cdot \text{mm}$$

$$A_s := \frac{0.85 \cdot f'_c \cdot a \cdot b_w + A_{sf} \cdot f_y}{f_y} = 41.655 \cdot \text{cm}^2$$

$$\rho_w := \frac{A_s}{b_w \cdot d} = 0.029$$

$$\rho_{w,\max} := \rho_{\max} + \rho_f = 0.034$$

$$\text{Concrete} := \begin{cases} \text{"is enough"} & \text{if } \rho_w \leq \rho_{w,\max} \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

Concrete = "is enough"

$$A_{s,\min} := \rho_{\min} \cdot b_w \cdot d$$

$$A_s := \max(A_s, A_{s,\min}) = 41.655 \cdot \text{cm}^2$$

$$6 \cdot \frac{\pi \cdot (32 \text{mm})^2}{4} = 48.255 \cdot \text{cm}^2$$

13. Shear Design

Safety provision

$$V_u \leq \phi V_n$$

where V_u = required shear strength

V_n = nominal shear strength

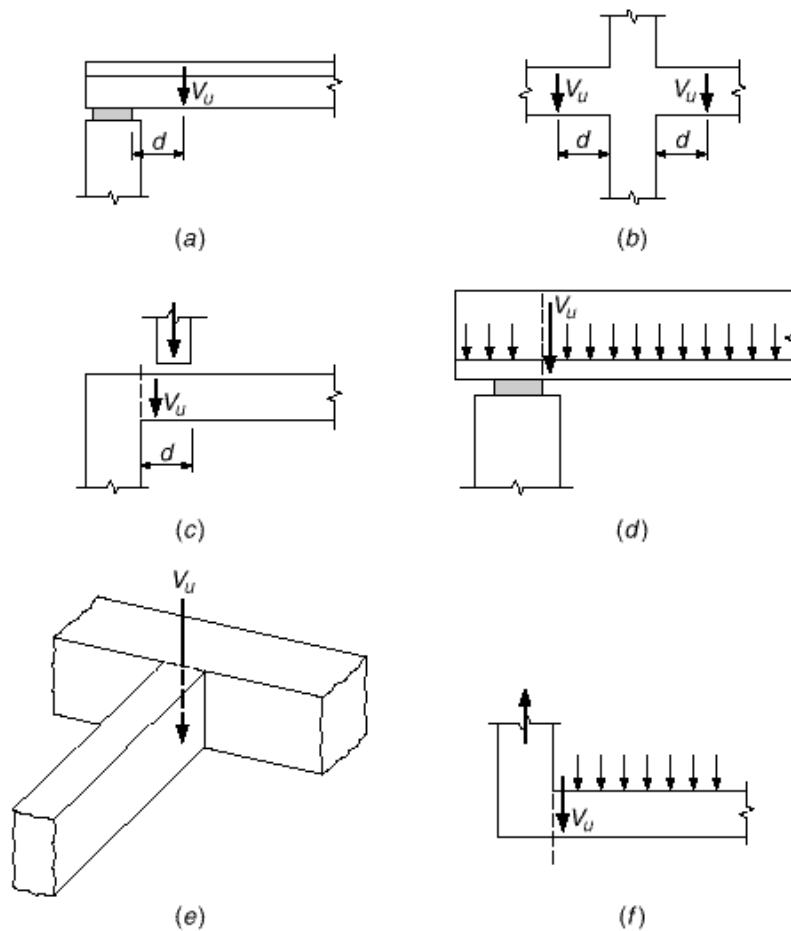
$\phi = 0.75$ is a strength reduction factor for shear

ϕV_n = design shear strength

Required shear strength

FIGURE 4.12

Location of critical section for shear design: (a) end-supported beam; (b) beam supported by columns; (c) concentrated load within d of the face of the support; (d) member loaded near the bottom; (e) beam supported by girder of similar depth; (f) beam supported by monolithic vertical element.



Nominal shear strength

$$V_n = V_c + V_s$$

where

V_c = concrete shear strength

V_s = steel shear strength

Concrete shear strength

$$V_c = 2 \cdot \sqrt{f'_c} \cdot b_w \cdot d \quad (\text{in psi})$$

$$V_c = 0.166 \cdot \sqrt{f'_c} \cdot b_w \cdot d \quad (\text{in MPa})$$

Steel shear strength

$$V_s = \frac{A_v \cdot f_y \cdot d}{s}$$

where

A_v = area of stirrup

f_y = yield strength of stirrup

s = spacing of stirrup

No required stirrups

$$V_u \leq \frac{\phi V_c}{2} \quad : \text{no stirrup is required}$$

$$\frac{\phi V_c}{2} \leq V_u \leq \phi V_c \quad : \text{stirrup is minimum}$$

$$V_u > \phi V_c \quad : \text{stirrup is required}$$

Minimum stirrups

$$A_{v,\min} = 0.75 \cdot \sqrt{f'_c} \cdot \frac{b_w \cdot s}{f_y} \geq 50 \cdot \frac{b_w \cdot s}{f_y} \quad (\text{in psi})$$

$$A_{v,\min} = 0.062 \cdot \sqrt{f'_c} \cdot \frac{b_w \cdot s}{f_y} \geq 0.345 \cdot \frac{b_w \cdot s}{f_y} \quad (\text{in MPa})$$

Maximum spacing of stirrup

$$s_{\max} = \frac{A_v \cdot f_y}{0.75 \cdot \sqrt{f_c} \cdot b_w} \leq \frac{A_v \cdot f_y}{50 \cdot b_w} \quad (\text{in psi})$$

$$s_{\max} = \frac{A_v \cdot f_y}{0.062 \cdot \sqrt{f_c} \cdot b_w} \leq \frac{A_v \cdot f_y}{0.345 \cdot b_w} \quad (\text{in MPa})$$

Case $V_s \leq 2 \cdot V_c$

$$s_{\max} = \frac{d}{2} \leq 24\text{in} = 600\text{mm}$$

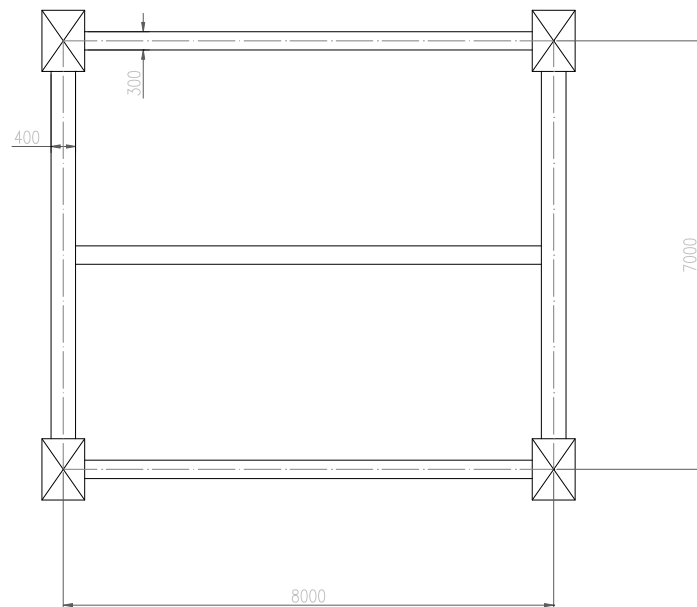
Case $2 \cdot V_c < V_s \leq 4 \cdot V_c$

$$s_{\max} = \frac{d}{4} \leq 12\text{in} = 300\text{mm}$$

Case $V_s > 4 \cdot V_c$

Concrete is not enough

Example 13.1



Materials

$$f_c := 25\text{MPa}$$

$$f_y := 390\text{MPa}$$

Live load for garage $LL := 6.00 \frac{\text{kN}}{\text{m}^2}$

Loads on slab

$$\text{Hardener} := 8\text{mm} \cdot 24 \frac{\text{kN}}{\text{m}^3} = 0.192 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$\text{Slab} := 200\text{mm} \cdot 25 \frac{\text{kN}}{\text{m}^3} = 5 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$\text{Mechanical} := 0.30 \frac{\text{kN}}{\text{m}^2}$$

$$DL := \text{Hardener} + \text{Slab} + \text{Mechanical} = 5.492 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$LL = 6 \cdot \frac{\text{kN}}{\text{m}^2}$$

Loads on beam

$$w_{\text{beam}} := 30\text{cm} \cdot (60\text{cm} - 200\text{mm}) \cdot 25 \frac{\text{kN}}{\text{m}^3} = 3 \cdot \frac{\text{kN}}{\text{m}}$$

$$w_{D,\text{slab}} := DL \cdot 3.5\text{m} = 19.222 \cdot \frac{\text{kN}}{\text{m}}$$

$$w_{L,\text{slab}} := LL \cdot 3.5\text{m} = 21 \cdot \frac{\text{kN}}{\text{m}}$$

$$w_D := w_{\text{beam}} + w_{D,\text{slab}} = 22.222 \cdot \frac{\text{kN}}{\text{m}}$$

$$w_L := w_{L,\text{slab}} = 21 \cdot \frac{\text{kN}}{\text{m}}$$

$$w_u := 1.2 \cdot w_D + 1.6 \cdot w_L = 60.266 \cdot \frac{\text{kN}}{\text{m}}$$

Shear

$$L := 8\text{m}$$

$$V_0 := \frac{w_u \cdot L}{2} = 241.066 \cdot \text{kN}$$

$$V(x) := V_0 - w_u \cdot x$$

Concrete shear strength

$$b_w := 300\text{mm} \quad d := 600\text{mm} - \left(40\text{mm} + 10\text{mm} + \frac{20\text{mm}}{2} \right) = 540\text{mm}$$

$$V_c := 0.166\text{MPa} \cdot \sqrt{\frac{f_c}{\text{MPa}}} \cdot b_w \cdot d = 134.46 \cdot \text{kN}$$

$$\phi := 0.75$$

Location of no stirrup zone

$$V_0 - w_u \cdot x = \frac{\phi V_c}{2} \quad x := \frac{V_0 - \frac{\phi V_c}{2}}{w_u} = 3.163 \text{ m}$$

Minimum stirrup

$$A_v := 2 \cdot \frac{\pi \cdot (10\text{mm})^2}{4} = 1.571 \cdot \text{cm}^2 \quad f_y := 390\text{MPa}$$

$$s_{\max} := \min \left(\frac{A_v \cdot f_y}{0.062\text{MPa} \cdot \sqrt{\frac{f_c}{\text{MPa}}} \cdot b_w}, \frac{A_v \cdot f_y}{0.345\text{MPa} \cdot b_w} \right) = 591.894 \cdot \text{mm}$$

$$s_{\max} := \text{Floor}(s_{\max}, 50\text{mm}) = 550 \cdot \text{mm}$$

$$V_{s,\min} := \frac{A_v \cdot f_y \cdot d}{s_{\max}} = 60.147 \cdot \text{kN}$$

Location of minimum stirrup zone

$$V_0 - w_u \cdot x = \phi \cdot (V_c + V_{s,\min}) \quad x := \frac{V_0 - \phi \cdot (V_c + V_{s,\min})}{w_u} = 1.578 \cdot \text{m}$$

Required spacing of stirrup

$$V_u := V_0 - w_u \cdot \left(\frac{400\text{mm}}{2} \right) = 229.012 \cdot \text{kN}$$

$$V_s := \frac{V_u}{\phi} - V_c = 170.89 \cdot \text{kN}$$

$$\text{Concrete} := \begin{cases} \text{"is enough"} & \text{if } V_s \leq 4 \cdot V_c \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

Concrete = "is enough"

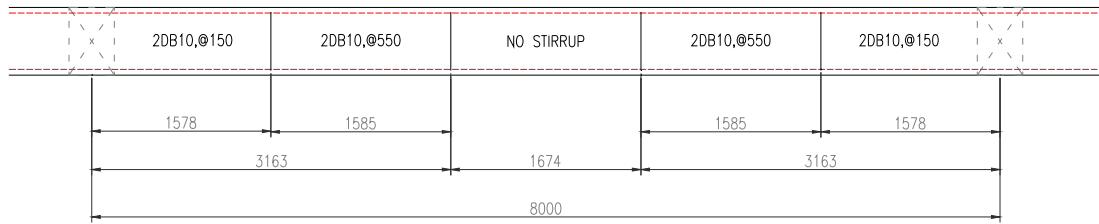
$$s := \frac{A_v \cdot f_y \cdot d}{V_s} = 193.581 \cdot \text{mm}$$

$$s_{\max,1} := s_{\max} = 550 \cdot \text{mm}$$

$$s_{\max,2} := \begin{cases} \min\left(\frac{d}{2}, 600\text{mm}\right) & \text{if } V_s \leq 2 \cdot V_c \\ \min\left(\frac{d}{4}, 300\text{mm}\right) & \text{otherwise} \end{cases} \quad s_{\max,2} = 270 \cdot \text{mm}$$

$$s := \text{Floor}(\min(s, s_{\max,1}, s_{\max,2}), 50\text{mm})$$

s = 150·mm



Example 13.2

Design of shear in support and midspan zones.

Stirrups in Support Zone

Required shear strength $V_u := V_0 - w_u \cdot \frac{400\text{mm}}{2} = 229.012 \cdot \text{kN}$

Concrete shear strength

$$V_c := 0.166 \text{MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}} \cdot b_w \cdot d = 134.46 \cdot \text{kN}$$

$$\phi := 0.75$$

$$\text{Stirrup} := \begin{cases} \text{"is minimum"} & \text{if } V_u \leq \phi \cdot V_c \\ \text{"is required"} & \text{otherwise} \end{cases}$$

Stirrup = "is required"

Required steel shear strength

$$V_s := \frac{V_u}{\phi} - V_c = 170.89 \cdot \text{kN}$$

$$\text{Concrete} := \begin{cases} \text{"is enough"} & \text{if } V_s \leq 4 \cdot V_c \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

Concrete = "is enough"

Spacing of stirrup

$$A_v := 2 \cdot \frac{\pi \cdot (10\text{mm})^2}{4} = 1.571 \cdot \text{cm}^2 \quad f_y := 390 \text{MPa}$$

$$s := \frac{A_v \cdot f_y \cdot d}{V_s} = 193.581 \cdot \text{mm}$$

$$s_{\max.1} := \min \left(\frac{A_v \cdot f_y}{0.062 \text{MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}} \cdot b_w}, \frac{A_v \cdot f_y}{0.345 \text{MPa} \cdot b_w} \right) = 591.894 \cdot \text{mm}$$

$$s_{\max.2} := \begin{cases} \min\left(\frac{d}{2}, 600\text{mm}\right) & \text{if } V_s \leq 2 \cdot V_c \\ \min\left(\frac{d}{4}, 300\text{mm}\right) & \text{otherwise} \end{cases} \quad s_{\max.2} = 270 \cdot \text{mm}$$

$$s := \text{Floor}(\min(s, s_{\max.1}, s_{\max.2}), 50\text{mm}) = 150 \cdot \text{mm}$$

Stirrups in Midspan Zone

Required shear strength $V_u := V_0 - w_u \cdot \frac{L}{4} = 120.533 \cdot \text{kN}$

$$\text{Stirrup} := \begin{cases} \text{"is minimum"} & \text{if } V_u \leq \phi \cdot V_c \\ \text{"is required"} & \text{otherwise} \end{cases} \quad \text{Stirrup} = \text{"is required"}$$

Required steel shear strength

$$V_s := \frac{V_u}{\phi} - V_c = 26.25 \cdot \text{kN}$$

$$\text{Concrete} := \begin{cases} \text{"is enough"} & \text{if } V_s \leq 4 \cdot V_c \\ \text{"is not enough"} & \text{otherwise} \end{cases} \quad \text{Concrete} = \text{"is enough"}$$

Spacing of stirrup

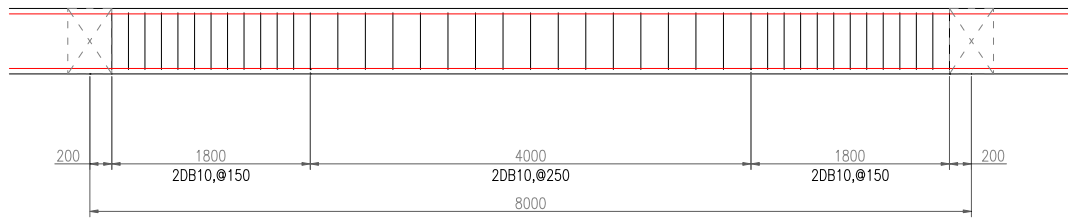
$$A_v := 2 \cdot \frac{\pi \cdot (10\text{mm})^2}{4} = 1.571 \cdot \text{cm}^2 \quad f_y := 390\text{MPa}$$

$$s := \frac{A_v \cdot f_y \cdot d}{V_s} = 1260.208 \cdot \text{mm}$$

$$s_{\max.1} := \min\left(\frac{A_v \cdot f_y}{0.062\text{MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}} \cdot b_w}, \frac{A_v \cdot f_y}{0.345\text{MPa} \cdot b_w}\right) = 591.894 \cdot \text{mm}$$

$$s_{\max.2} := \begin{cases} \min\left(\frac{d}{2}, 600\text{mm}\right) & \text{if } V_s \leq 2 \cdot V_c \\ \min\left(\frac{d}{4}, 300\text{mm}\right) & \text{otherwise} \end{cases} \quad s_{\max.2} = 270 \cdot \text{mm}$$

$$s := \text{Floor}(\min(s, s_{\max.1}, s_{\max.2}), 50\text{mm}) = 250 \cdot \text{mm}$$



14. Column Design

Type of columns (by design method)

1. Axially loaded columns

$$e = \frac{M}{P} = 0$$

2. Eccentric columns

$$e = \frac{M}{P} \neq 0$$

- 2.1. Short columns (without buckling)

$$P_u, M_u$$

- 2.2. Long (slender) columns (with buckling)

$$P_u, M_u \cdot \delta_{ns}$$

1. Axially Loaded Columns

Safety provision

$$P_u \leq \phi P_{n,max}$$

where P_u = axial load on column
 $\phi P_{n,max}$ = design axial strength

For tied columns

$$\phi P_{n,max} = 0.80 \cdot \phi \cdot [0.85 \cdot f'_c \cdot (A_g - A_{st}) + f_y \cdot A_{st}]$$

with $\phi = 0.65$

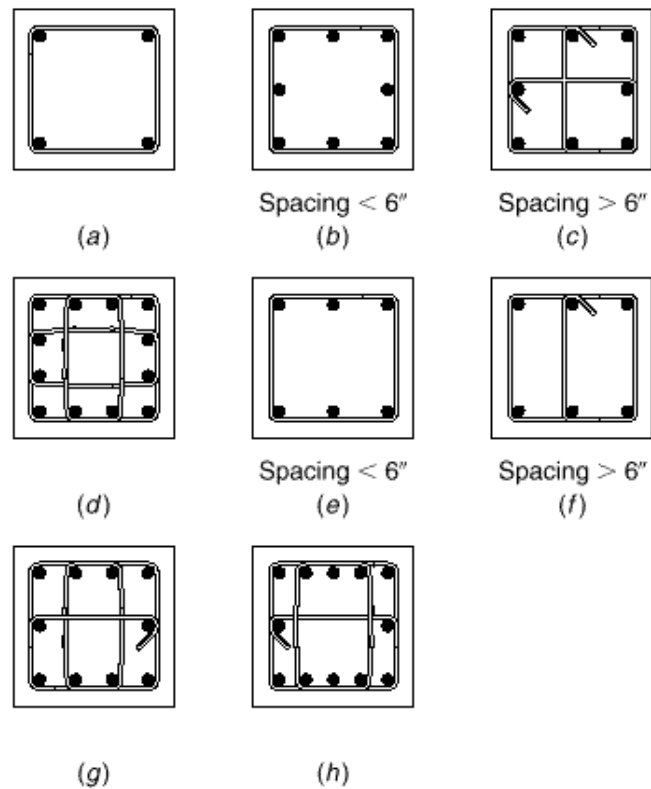
For spirally reinforced columns

$$\phi P_{n,max} = 0.85 \cdot \phi \cdot [0.85 \cdot f'_c \cdot (A_g - A_{st}) + f_y \cdot A_{st}]$$

with $\phi = 0.70$

where A_g = area of gross section
 A_{st} = area of steel reinforcements
 $A_g - A_{st} = A_c$ is an area of concrete section

FIGURE 8.2
Tie arrangements for square
and rectangular columns.



For tied columns

Diameter of tie

$$D_v = 10\text{mm} \quad \text{for} \quad D \leq 32\text{mm}$$

$$D_v = 12\text{mm} \quad \text{for} \quad D > 32\text{mm}$$

Spacing of tie

$$s \leq 48D_v \quad s \leq 16D \quad s \leq b$$

For spirally reinforced columns

$$\text{Diameter of spiral} \quad D_v \geq 10\text{mm}$$

$$\text{Clear spacing} \quad 25\text{mm} \leq s \leq 75\text{mm}$$

Column steel ratio

$$\rho_g = \frac{A_{st}}{A_g} = 1\% \text{..} 8\%$$

Determination of Concrete Section

$$A_g = \frac{\frac{P_u}{0.80 \cdot \phi}}{0.85 \cdot f'_c \cdot (1 - \rho_g) + f_y \cdot \rho_g}$$

Determination of Steel Area

$$A_{st} = \frac{\frac{P_u}{0.80 \cdot \phi} - 0.85 \cdot f'_c \cdot A_g}{-0.85 \cdot f'_c + f_y}$$

Example 14.1

Tributary area	B := 4m	L := 6m
Thickness of slab	t := 120mm	
Section of beam B1	b := 250mm	h := 500mm
Section of beam B2	b := 200mm	h := 350mm
Live load for lab	LL := 3.00 $\frac{\text{kN}}{\text{m}^2}$	
Materials	$f'_c := 25\text{MPa}$	$f_y := 390\text{MPa}$

Solution

Loads on slab

$$\text{Cover} := 50\text{mm} \cdot 22 \frac{\text{kN}}{\text{m}^3}$$

$$\text{Slab} := 120\text{mm} \cdot 25 \frac{\text{kN}}{\text{m}^3}$$

$$\text{Ceiling} := 0.40 \frac{\text{kN}}{\text{m}^2}$$

$$\text{Mechanical} := 0.20 \frac{\text{kN}}{\text{m}^2}$$

$$\text{Partition} := 1.00 \frac{\text{kN}}{\text{m}^2}$$

$$\text{DL} := \text{Cover} + \text{Slab} + \text{Ceiling} + \text{Mechanical} + \text{Partition} = 5.7 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$\text{LL} = 3 \cdot \frac{\text{kN}}{\text{m}^2}$$

Reduction of live load

Tributary area	$A_T := B \cdot L = 24 \text{ m}^2$
For interior column	$K_{LL} := 4$
Influence area	$A_I := K_{LL} \cdot A_T = 96 \text{ m}^2$
Live load reduction factor	$\alpha_{LL} := 0.25 + \frac{4.572}{\sqrt{\frac{A_I}{\text{m}^2}}} = 0.717$
Reduced live load	$LL_0 := LL \cdot \alpha_{LL} = 2.15 \cdot \frac{\text{kN}}{\text{m}^2}$

Loads of wall

$$\text{Void} := 30\text{mm} \cdot 30\text{mm} \cdot 190\text{mm} \cdot 4$$

$$\text{Brick}_{\text{hollow}.10} := \left(120\text{mm} - \text{Void} \cdot \frac{55}{1\text{m}^2} \right) \cdot 20 \frac{\text{kN}}{\text{m}^3} = 1.648 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$\text{Brick}_{\text{hollow}.20} := \left(220\text{mm} - \text{Void} \cdot \frac{110}{1\text{m}^2} \right) \cdot 20 \frac{\text{kN}}{\text{m}^3} = 2.895 \cdot \frac{\text{kN}}{\text{m}^2}$$

Loads on column

$$P_{D.\text{slab}} := DL \cdot B \cdot L = 136.8 \cdot \text{kN}$$

$$P_{L.\text{slab}} := LL \cdot B \cdot L = 72 \cdot \text{kN}$$

$$P_{B1} := 25\text{cm} \cdot (50\text{cm} - 120\text{mm}) \cdot 25 \frac{\text{kN}}{\text{m}^3} \cdot L = 14.25 \cdot \text{kN}$$

$$P_{B2} := 20\text{cm} \cdot (35\text{cm} - 120\text{mm}) \cdot 25 \frac{\text{kN}}{\text{m}^3} \cdot B = 4.6 \cdot \text{kN}$$

$$P_{\text{wall}.1} := \text{Brick}_{\text{hollow}.10} \cdot (3.5\text{m} - 50\text{cm}) \cdot L = 29.657 \cdot \text{kN}$$

$$P_{\text{wall}.2} := \text{Brick}_{\text{hollow}.10} \cdot (3.5\text{m} - 35\text{cm}) \cdot B = 20.76 \cdot \text{kN}$$

$$\text{Number of floors} \quad n := 6$$

$$P_D := (P_{D.\text{slab}} + P_{B1} + P_{B2} + P_{\text{wall}.1} + P_{\text{wall}.2}) \cdot 1.05 \cdot n = 1298.219 \cdot \text{kN}$$

$$P_L := P_{L.\text{slab}} \cdot n = 432 \cdot \text{kN} \quad \text{SW} = (5\% \dots 7\%) \cdot P_D$$

$$\frac{P_D + P_L}{B \cdot L \cdot n} = 12.015 \cdot \frac{\text{kN}}{\text{m}^2} \quad \frac{P_L}{P_D + P_L} = 24.968 \cdot \%$$

$$P_u := 1.2 \cdot P_D + 1.6 \cdot P_L = 2249.063 \cdot \text{kN}$$

Determination of column section

Assume $\rho_g := 0.03$ $k = \frac{b}{h}$ $k := \frac{300}{500}$

$$\phi := 0.65$$

$$A_g := \frac{\frac{P_u}{0.80 \cdot \phi}}{0.85 \cdot f_c \cdot (1 - \rho_g) + f_y \cdot \rho_g} \quad A_g = 1338.529 \cdot \text{cm}^2$$

$$h := \sqrt{\frac{A_g}{k}} = 472.322 \cdot \text{mm} \quad b := k \cdot h = 283.393 \cdot \text{mm}$$

$$h := \text{Ceil}(h, 50\text{mm}) = 500 \cdot \text{mm} \quad b := \text{Ceil}(b, 50\text{mm}) = 300 \cdot \text{mm}$$

$$\begin{pmatrix} b \\ h \end{pmatrix} = \begin{pmatrix} 300 \\ 500 \end{pmatrix} \cdot \text{mm} \quad A_g := b \cdot h = 1500 \cdot \text{cm}^2$$

Determination of steel area

$$A_{st} := \frac{\frac{P_u}{0.80 \cdot \phi} - 0.85 \cdot f_c \cdot A_g}{-0.85 \cdot f_c + f_y} \quad A_{st} = 30.851 \cdot \text{cm}^2$$

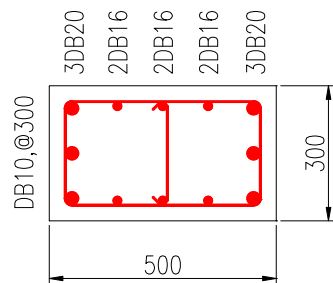
$$6 \cdot \frac{\pi \cdot (20\text{mm})^2}{4} + 6 \cdot \frac{\pi \cdot (16\text{mm})^2}{4} = 30.913 \cdot \text{cm}^2$$

Stirrups

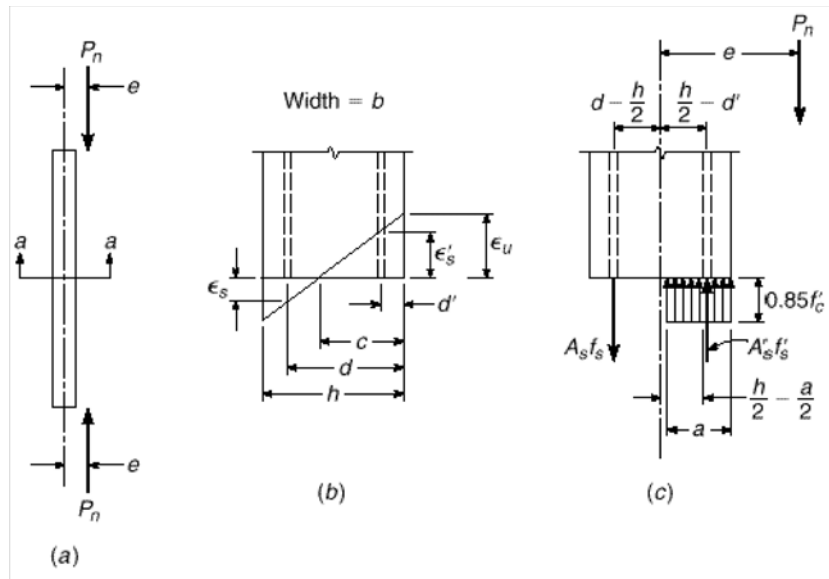
Main bars $D := 20\text{mm}$

Stirrup dia. $D_v := 10\text{mm}$

Spacing of tie $s := \min(16 \cdot D, 48 \cdot D_v, b) = 300 \cdot \text{mm}$



2. Short Columns



Safety provision

$$P_u \leq \phi P_n$$

$$M_u \leq \phi M_n$$

Equilibrium in forces $\sum X = 0$

$$P_n = C + C_s - T$$

$$P_n = 0.85 \cdot f'_c \cdot a \cdot b + A'_s \cdot f'_s - A_s \cdot f_s$$

Equilibrium in moments $\sum M = 0$

$$M_n = P_n \cdot e = C \cdot \left(\frac{h}{2} - \frac{a}{2} \right) + C_s \cdot \left(\frac{h}{2} - d' \right) + T \cdot \left(d - \frac{h}{2} \right)$$

$$M_n = P_n \cdot e = 0.85 \cdot f'_c \cdot a \cdot b \cdot \left(\frac{h}{2} - \frac{a}{2} \right) + A'_s \cdot f'_s \cdot \left(\frac{h}{2} - d' \right) + A_s \cdot f_s \cdot \left(d - \frac{h}{2} \right)$$

Conditions of strain compatibility

$$\frac{\epsilon_s}{\epsilon_u} = \frac{d - c}{c}$$

$$\epsilon_s = \epsilon_u \cdot \frac{d - c}{c}$$

$$f_s = E_s \cdot \epsilon_s = E_s \cdot \epsilon_u \cdot \frac{d - c}{c}$$

$$\frac{\epsilon'_s}{\epsilon_u} = \frac{c - d'}{c}$$

$$\epsilon'_s = \epsilon_u \cdot \frac{c - d'}{c}$$

$$f'_s = E_s \cdot \epsilon'_s = E_s \cdot \epsilon_u \cdot \frac{c - d'}{c}$$

Unknowns = 5 : a, A_s, A'_s, f_s, f'_s

Equations = 4 : $\sum X = 0$ $\sum M = 0$ 2 conditions of strain compatibility

Case of symmetrical columns: $A_s = A'_s$

Case of unsymmetrical columns: $f_s = f_y$

A. Interaction Diagram for Column Strength

Interaction diagram is a graph of parametric function, where

Abscissa : $M_n(a)$

Ordinate: $P_n(a)$

B. Determination of Steel Area

Given: $M_u, P_u, b \times h, f'_c, f_y$

Find: $A_s = A'_s$

Answer: $A_s = A_{sN}(a) = A_{sM}(a)$

$$A_{sN}(a) = \frac{\frac{P_u}{\phi} - 0.85 \cdot f'_c \cdot a \cdot b}{f_s - f'_s}$$

$$A_{sM}(a) = \frac{\frac{M_u}{\phi} - 0.85 \cdot f'_c \cdot a \cdot b \cdot \left(\frac{h}{2} - \frac{a}{2}\right)}{f'_s \cdot \left(\frac{h}{2} - d'\right) + f_s \cdot \left(d - \frac{h}{2}\right)}$$

$$f'_s(a) = E_s \cdot \epsilon_u \cdot \frac{c - d'}{c} \leq f_y$$

$$f_s(a) = E_s \cdot \epsilon_u \cdot \frac{d - c}{c} \leq f_y$$

Example 14.2

Construction of interaction diagram for column strength.

Concrete dimension

$$b := 500\text{mm} \quad h := 200\text{mm}$$

Steel reinforcements

$$A_s := 5 \cdot \frac{\pi \cdot (16\text{mm})^2}{4} = 10.053 \cdot \text{cm}^2$$

$$A'_s := A_s = 10.053 \cdot \text{cm}^2$$

$$d' := 30\text{mm} + 6\text{mm} + \frac{16\text{mm}}{2} = 44\text{mm}$$

$$d := h - d' = 156\text{mm}$$

Materials

$$f_c := 25\text{MPa}$$

$$f_y := 390\text{MPa}$$

Solution

Case of axially loaded column

$$A_g := b \cdot h$$

$$A_{st} := A_s + A'_s$$

$$\phi := 0.65$$

$$\phi P_{n,\max} := 0.80 \cdot \phi \left[0.85 \cdot f_c \cdot (A_g - A_{st}) + f_y \cdot A_{st} \right] = 1490.536 \cdot \text{kN}$$

Case of eccentric column

$$\beta_1 := 0.65 \max \left[\left(0.85 - 0.05 \cdot \frac{f_c - 27.6\text{MPa}}{6.9\text{MPa}} \right) \min 0.85 \right] = 0.85$$

$$c(a) := \frac{a}{\beta_1}$$

$$E_s := 2 \cdot 10^5 \text{MPa} \quad \epsilon_u := 0.003 \quad d_t := d$$

$$f_s(a) := \min \left(E_s \cdot \epsilon_u \cdot \frac{d - c(a)}{c(a)}, f_y \right)$$

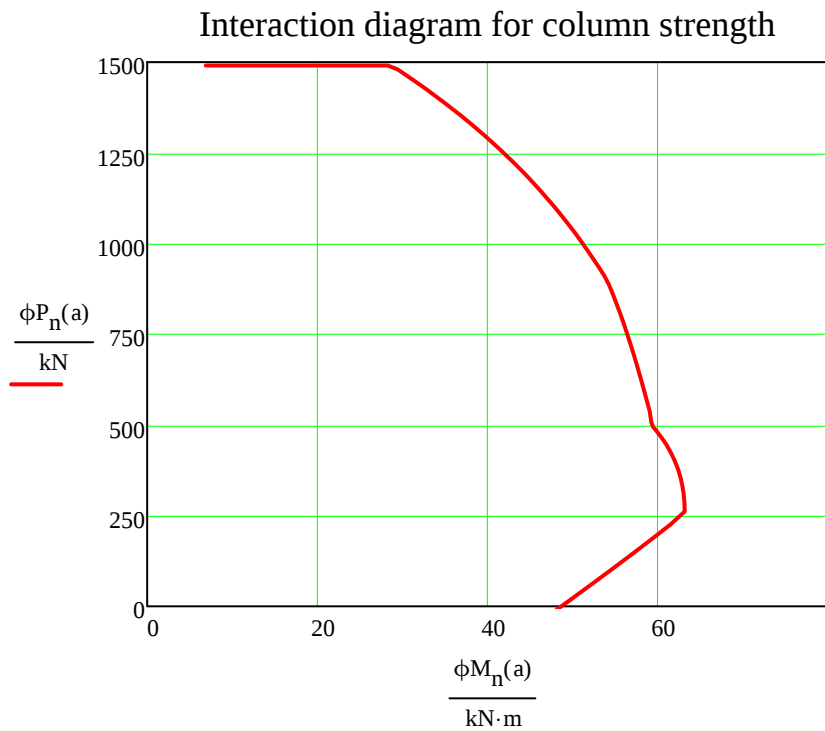
$$f'_s(a) := \min \left(E_s \cdot \epsilon_u \cdot \frac{c(a) - d'}{c(a)}, f_y \right)$$

$$\phi(a) := \begin{cases} \epsilon_t \leftarrow \epsilon_u \cdot \frac{d_t - c(a)}{c(a)} \\ \phi \leftarrow 0.65 \max \left[\left(\frac{1.45 + 250 \cdot \epsilon_t}{3} \right) \min 0.90 \right] \end{cases}$$

$$\phi P_n(a) := \min \left[\phi(a) \cdot (0.85 \cdot f'_c \cdot a \cdot b + A'_s \cdot f'_s(a) - A_s \cdot f_s(a)), \phi P_{n,\max} \right]$$

$$\phi M_n(a) := \phi(a) \cdot \left[0.85 \cdot f'_c \cdot a \cdot b \cdot \left(\frac{h}{2} - \frac{a}{2} \right) + A'_s \cdot f'_s(a) \cdot \left(\frac{h}{2} - d' \right) + A_s \cdot f_s(a) \cdot \left(d - \frac{h}{2} \right) \right]$$

$$a := 0, \frac{h}{100} \dots h$$



Example 14.3

Determination of steel area.

Required strength

$$P_u := 1152.27 \text{ kN}$$

$$M_u := 42.64 \text{ kN}\cdot\text{m}$$

Concrete dimension

$$b := 500 \text{ mm} \quad h := 200 \text{ mm}$$

Materials

$$f'_c := 25 \text{ MPa}$$

$$f_y := 390 \text{ MPa}$$

Concrete cover to main bars

$$c_c := 30 \text{ mm} + 6 \text{ mm} + \frac{16 \text{ mm}}{2}$$

Solution

Location of steel re-bars

$$d' := c_c = 44 \cdot \text{mm}$$

$$d := h - c_c = 156 \cdot \text{mm}$$

Case of axially loaded column

$$A_g := b \cdot h \quad \phi := 0.65$$

$$A_{st} := \frac{\frac{P_u}{0.80 \cdot \phi} - 0.85 \cdot f'_c \cdot A_g}{-0.85 \cdot f'_c + f_y} = 2.465 \cdot \text{cm}^2$$

Case of eccentric column

$$\beta_1 := 0.65 \max \left[\left(0.85 - 0.05 \cdot \frac{f'_c - 27.6 \text{MPa}}{6.9 \text{MPa}} \right) \min 0.85 \right] = 0.85$$

$$c(a) := \frac{a}{\beta_1}$$

$$E_s := 2 \cdot 10^5 \text{MPa} \quad \epsilon_u := 0.003 \quad d_t := d$$

$$f_s(a) := \min \left(E_s \cdot \epsilon_u \cdot \frac{d - c(a)}{c(a)}, f_y \right)$$

$$f'_s(a) := \min \left(E_s \cdot \epsilon_u \cdot \frac{c(a) - d'}{c(a)}, f_y \right)$$

$$\phi(a) := \begin{cases} \epsilon_t \leftarrow \epsilon_u \cdot \frac{d_t - c(a)}{c(a)} \\ \phi \leftarrow 0.65 \max \left[\left(\frac{1.45 + 250 \cdot \epsilon_t}{3} \right) \min 0.90 \right] \end{cases}$$

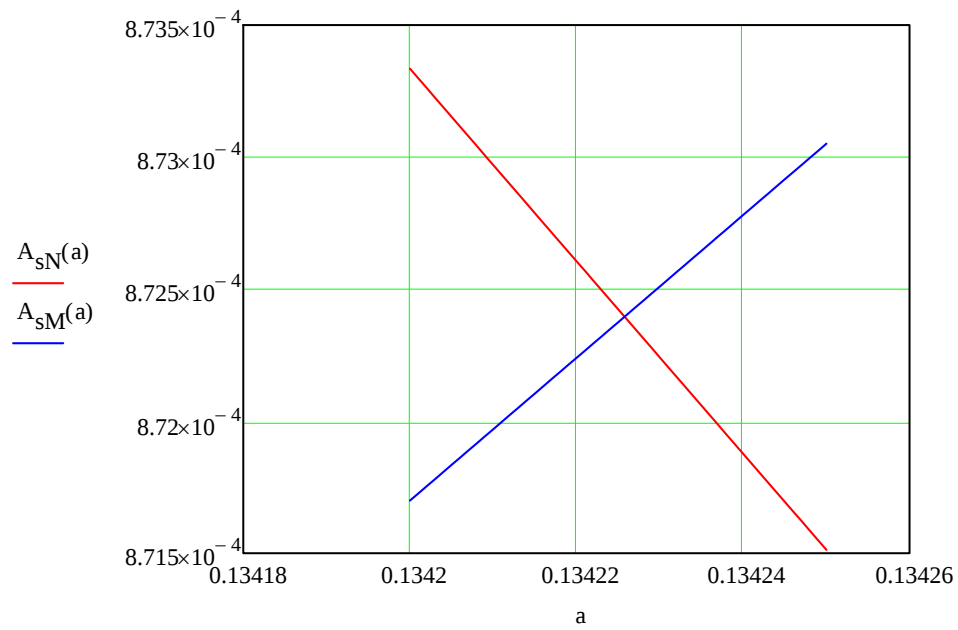
Graphical solution

$$A_{sN}(a) := \frac{\frac{P_u}{\phi(a)} - 0.85 \cdot f'_c \cdot a \cdot b}{f'_s(a) - f_s(a)}$$

$$A_{sM}(a) := \frac{\frac{M_u}{\phi(a)} - 0.85 \cdot f'_c \cdot a \cdot b \cdot \left(\frac{h}{2} - \frac{a}{2} \right)}{f'_s(a) \cdot \left(\frac{h}{2} - d' \right) + f_s(a) \cdot \left(d - \frac{h}{2} \right)}$$

$$a1 := 134.2 \text{mm} \quad a2 := 134.25 \text{mm}$$

$$a := a1, a1 + \frac{a2 - a1}{50} .. a2$$



$$a := 134.23 \text{ mm}$$

$$A_{sN}(a) = 8.722 \cdot \text{cm}^2$$

$$A_{sM}(a) = 8.725 \cdot \text{cm}^2$$

$$A_s := \frac{A_{sN}(a) + A_{sM}(a)}{2} = 8.724 \cdot \text{cm}^2$$

$$5 \cdot \frac{\pi \cdot (16 \text{ mm})^2}{4} = 10.053 \cdot \text{cm}^2$$

Analytical solution

ORIGIN := 1

```

Asteel(No) :=
  k ← 1
  for a ∈ cC, cC +  $\frac{h}{No}$  .. h
    f ← f's(a) - fs(a)
    (continue) if f = 0
    
$$A_{sN} \leftarrow \frac{\frac{P_u}{\phi(a)} - 0.85 \cdot f'_C \cdot a \cdot b}{f}$$

    (continue) if AsN ≤ 0
    fd ← f's(a) ·  $\left(\frac{h}{2} - d'\right)$  + fs(a) ·  $\left(d - \frac{h}{2}\right)$ 
    (continue) if fd = 0
    
$$A_{sM} \leftarrow \frac{\frac{M_u}{\phi(a)} - 0.85 \cdot f'_C \cdot a \cdot b \cdot \left(\frac{h}{2} - \frac{a}{2}\right)}{fd}$$

    (continue) if AsM ≤ 0
    
$$Z^{(k)} \leftarrow \begin{pmatrix} \frac{a}{h} \\ \frac{A_{sN}}{A_g} \\ \frac{A_{sM}}{A_g} \\ \left| \frac{A_{sN} - A_{sM}}{A_g} \right| \end{pmatrix}$$

    k ← k + 1
  csort(ZT, 4)

```

Z := Asteel(5000)

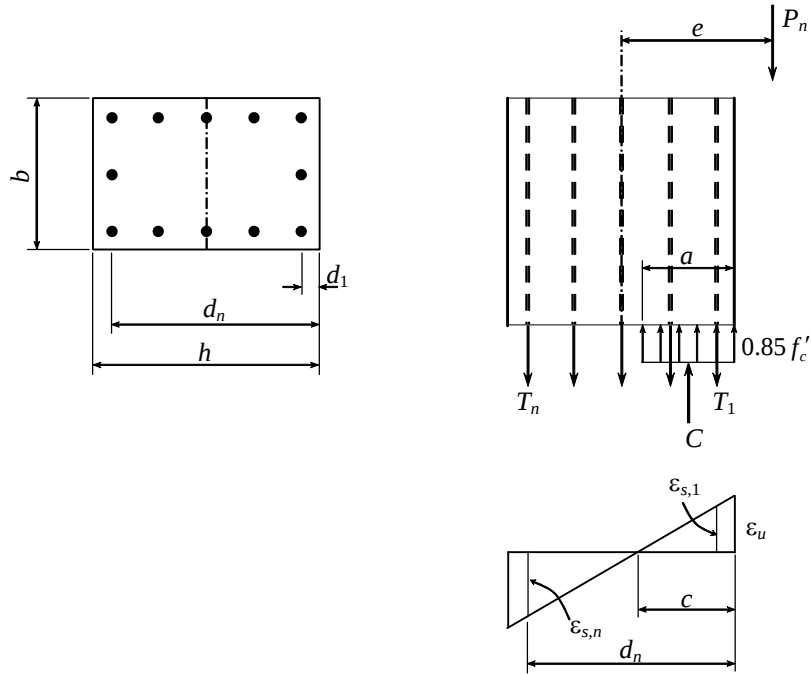
rows(Z) = 2046

a := Z_{1,1} · h = 134.24 · mm

A_{sN} := Z_{1,2} · A_g = 8.719 · cm² A_{sM} := Z_{1,3} · A_g = 8.728 · cm²

A_s := $\frac{A_{sN} + A_{sM}}{2}$ = 8.723 · cm²

C. Case of Distributed Reinforcements



រូប ៣.១. សសរចាក់ផ្គិត ដែលមានដែកពង្រាយច្រើនជួរ
ជុំវិញមុខកាត់បេតុង

Equilibrium in forces $\sum X = 0$

$$P_n = C - \sum_{i=1}^n T_i = 0.85 \cdot f'_c \cdot a \cdot b - \sum_{i=1}^n (A_{s,i} \cdot f_{s,i})$$

Equilibrium in moments $\sum M = 0$

$$M_n = P_n \cdot e = C \cdot \left(\frac{h}{2} - \frac{a}{2} \right) + \sum_{i=1}^n \left[T_i \cdot \left(d_i - \frac{h}{2} \right) \right]$$

$$M_n = 0.85 \cdot f'_c \cdot a \cdot b \cdot \left(\frac{h}{2} - \frac{a}{2} \right) + \sum_{i=1}^n \left[A_{s,i} \cdot f_{s,i} \cdot \left(d_i - \frac{h}{2} \right) \right]$$

Conditions of strain compatibility

$$\frac{\epsilon_{s,i}}{\epsilon_u} = \frac{d_i - c}{c}$$

$$\epsilon_{s,i} = \epsilon_u \cdot \frac{d_i - c}{c}$$

$$f_{s,i} = E_s \cdot \epsilon_{s,i} = E_s \cdot \epsilon_u \cdot \frac{d_i - c}{c}$$

Example 14.4

Checking for column strength.

Required strength $P_u := 13994.6 \text{ kN}$

$M_u := 57.53 \text{ kN}\cdot\text{m}$

Materials $f_c := 35 \text{ MPa}$

$f_y := 390 \text{ MPa}$

Solution

Determination of Concrete Section

Case of axially loaded column

$\phi := 0.65$

Assume $\rho_g := 0.04$

$$A_g := \frac{\frac{P_u}{0.80 \cdot \phi}}{0.85 \cdot f_c \cdot (1 - \rho_g) + f_y \cdot \rho_g} = 6094.36 \cdot \text{cm}^2$$

Aspect ratio of column section $\lambda = \frac{b}{h} \quad \lambda := 1$

$$h := \sqrt{\frac{A_g}{\lambda}} = 780.664 \cdot \text{mm}$$

$$b := \lambda \cdot h = 780.664 \cdot \text{mm}$$

$$h := \text{Ceil}(h, 50 \text{ mm})$$

$$b := \text{Ceil}(b, 50 \text{ mm})$$

$$\begin{pmatrix} b \\ h \end{pmatrix} = \begin{pmatrix} 800 \\ 800 \end{pmatrix} \cdot \text{mm}$$

$$A_g := b \cdot h = 6400 \cdot \text{cm}^2$$

Steel area

$$A_{st} := \frac{\frac{P_u}{0.80 \cdot \phi} - 0.85 \cdot f'_c \cdot A_g}{-0.85 \cdot f'_c + f_y} \quad A_{st} = 218.534 \cdot \text{cm}^2$$

$$(4 + 7 \cdot 4) \cdot \frac{\pi \cdot (25\text{mm})^2}{4} + (4 + 5 \cdot 4) \cdot \frac{\pi \cdot (20\text{mm})^2}{4} = 232.478 \cdot \text{cm}^2$$

$$\text{Spacing} \quad \frac{800\text{mm} - 50\text{mm} \cdot 2}{8} = 87.5 \cdot \text{mm}$$

Interaction Diagram for Column Strength

Distribution of reinforcements

$$\text{Bars} := \begin{pmatrix} 25 & 25 & 25 & 25 & 25 & 25 & 25 & 25 & 25 \\ 25 & 20 & 20 & 20 & 20 & 20 & 20 & 20 & 25 \\ 25 & 20 & 0 & 0 & 0 & 0 & 0 & 20 & 25 \\ 25 & 20 & 0 & 0 & 0 & 0 & 0 & 20 & 25 \\ 25 & 20 & 0 & 0 & 0 & 0 & 0 & 20 & 25 \\ 25 & 20 & 0 & 0 & 0 & 0 & 0 & 20 & 25 \\ 25 & 20 & 0 & 0 & 0 & 0 & 0 & 20 & 25 \\ 25 & 20 & 20 & 20 & 20 & 20 & 20 & 20 & 25 \\ 25 & 25 & 25 & 25 & 25 & 25 & 25 & 25 & 25 \end{pmatrix} \text{mm}$$

$$\text{Number of reinforcement rows} \quad n := \text{cols}(\text{Bars}) = 9$$

Steel area

$$A_{s0} := \frac{\pi \cdot \text{Bars}^2}{4}$$

$$i := 1 \dots n \quad A_{s_i} := \sum A_{s0}^{(i)}$$

$$A_{st} := \sum A_s \quad A_{st} = 232.478 \cdot \text{cm}^2$$

Location of reinforcement rows

$$\text{Concrete cover} \quad \text{Cover} := 30\text{mm} + 10\text{mm} = 40 \cdot \text{mm}$$

$$d_1 := \text{Cover} + \frac{\text{Bars}_{1,n}}{2} = 52.5 \cdot \text{mm} \quad \Delta S := \frac{h - d_1 \cdot 2}{n - 1} = 86.875 \cdot \text{mm}$$

$$i := 2 \dots n \quad d_i := d_{i-1} + \Delta S$$

$$\text{reverse}(d)^T = (747.5 \quad 660.63 \quad 573.75 \quad 486.88 \quad 400 \quad 313.13 \quad 226.25 \quad 139.38 \quad 52.5) \cdot \text{mm}$$

Case of axially loaded column

$$\phi P_{n,max} := 0.80 \cdot \phi \cdot \left[0.85 \cdot f'_c \cdot (A_g - A_{st}) + f_y \cdot A_{st} \right]$$

$$\phi P_{n,max} = 14255.808 \cdot \text{kN}$$

Case of eccentric column

$$\beta_1 := 0.65 \max \left[\left(0.85 - 0.05 \cdot \frac{f'_c - 27.6 \text{MPa}}{6.9 \text{MPa}} \right) \min 0.85 \right] = 0.796$$

$$c(a) := \frac{a}{\beta_1}$$

$$f_s(i, a) := \begin{cases} \epsilon_s \leftarrow \epsilon_u \cdot \frac{d_i - c(a)}{c(a)} \\ \text{sign}(\epsilon_s) \cdot \min(E_s \cdot |\epsilon_s|, f_y) \end{cases}$$

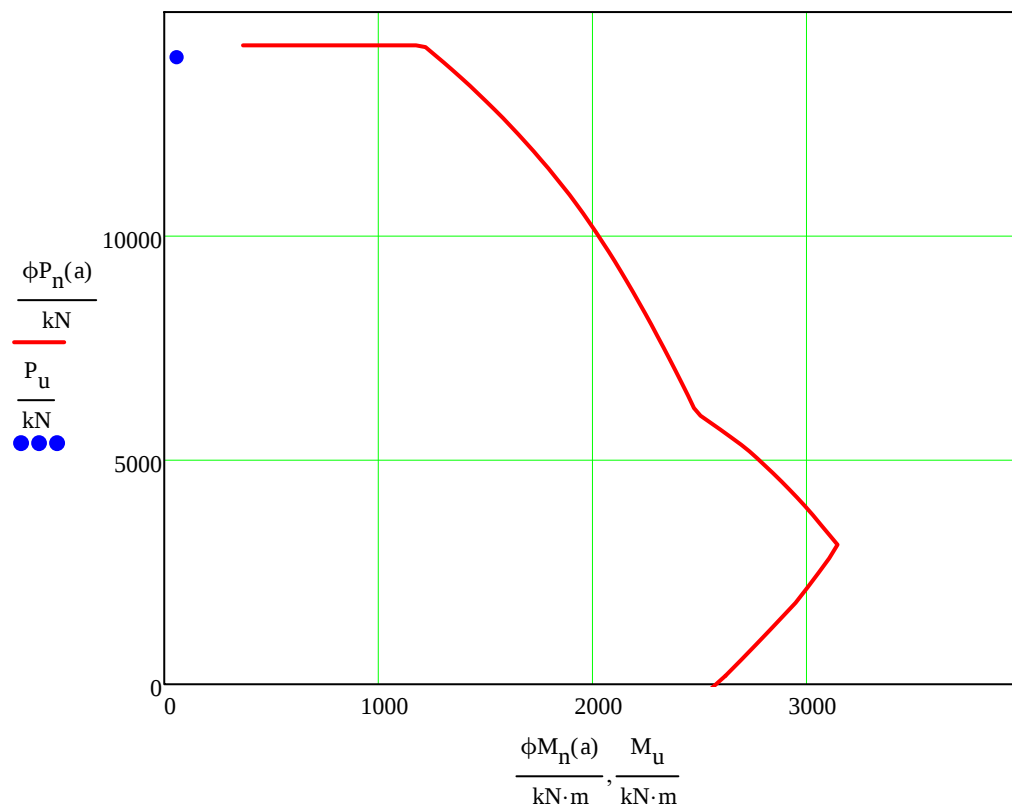
$$d_t := \max(d) = 747.5 \cdot \text{mm}$$

$$\phi(a) := \begin{cases} \epsilon_t \leftarrow \epsilon_u \cdot \frac{d_t - c(a)}{c(a)} \\ \phi \leftarrow 0.65 \max \left[\left(\frac{1.45 + 250 \cdot \epsilon_t}{3} \right) \min 0.9 \right] \end{cases}$$

$$\phi P_n(a) := \min \left[\phi(a) \cdot \left[0.85 \cdot f'_c \cdot a \cdot b - \sum_{i=1}^n \left(A_{s_i} \cdot f_s(i, a) \right) \right], \phi P_{n,max} \right]$$

$$\phi M_n(a) := \phi(a) \cdot \left[0.85 \cdot f'_c \cdot a \cdot b \cdot \left(\frac{h}{2} - \frac{a}{2} \right) + \sum_{i=1}^n \left[A_{s_i} \cdot f_s(i, a) \cdot \left(d_i - \frac{h}{2} \right) \right] \right]$$

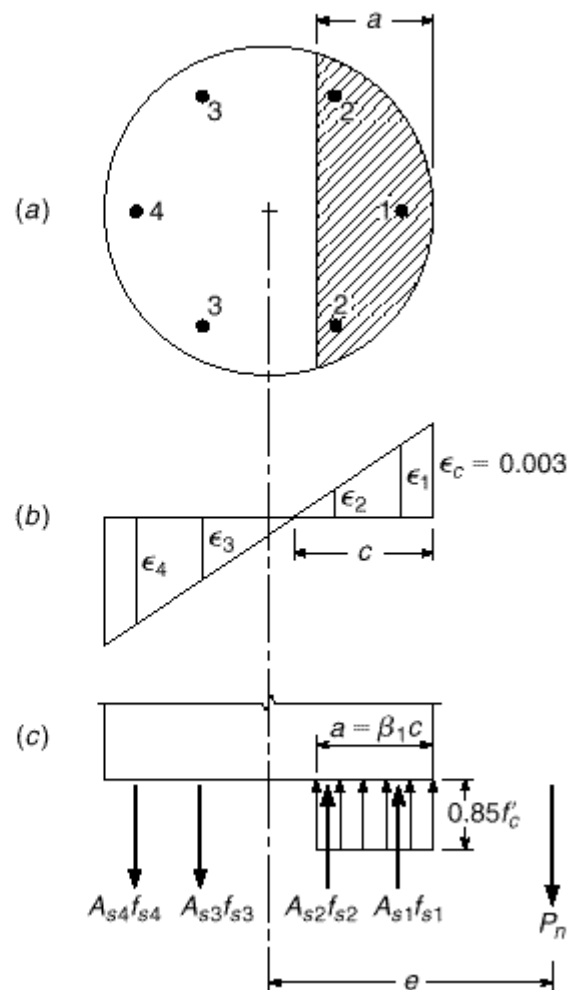
$$a := 0, \frac{h}{100} \dots h$$



D. Design of Circular Columns

FIGURE 8.13

Circular column with compression plus bending.



Symbols

- n_s = number of re-bars
 D_c = column diameter
 D_s = diameter of re-bar circle

Location of steel re-bar

$$d_i = r_c - r_s \cdot \cos(\alpha_{s,i}) \quad r_c = \frac{D_c}{2} \quad r_s = \frac{D_s}{2}$$

$$\alpha_{s,i} = \frac{2 \cdot \pi}{n_s} \cdot (i - 1)$$

Depth of compression concrete

$$\alpha = \arccos\left(\frac{r_c - a}{r_c}\right)$$

Area and centroid of compression concrete

$$A_{\text{sector}} = \frac{1}{2} \times \text{Radius} \times \text{Arch} = \frac{1}{2} \times r_c \times (r_c \cdot 2 \cdot \alpha) = r_c^2 \cdot \alpha$$

$$x_1 = \frac{2}{3} \cdot r_c \cdot \frac{\sin(\alpha)}{\alpha}$$

$$A_{\text{triangle}} = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times 2 \cdot r_c \cdot \sin(\alpha) \times r_c \cdot \cos(\alpha) = r_c^2 \cdot \sin(\alpha) \cdot \cos(\alpha)$$

$$x_2 = \frac{2}{3} \cdot r_c \cdot \cos(\alpha)$$

$$A_c = A_{\text{segment}} = A_{\text{sector}} - A_{\text{triangle}} = r_c^2 \cdot (\alpha - \sin(\alpha) \cdot \cos(\alpha))$$

$$x_c = \frac{A_{\text{sector}} \cdot x_1 - A_{\text{triangle}} \cdot x_2}{A_c} = \frac{2}{3} \cdot r_c \cdot \frac{\sin(\alpha) - \sin(\alpha) \cdot \cos(\alpha)^2}{\alpha - \sin(\alpha) \cdot \cos(\alpha)}$$

$$x_c = \frac{2 \cdot r_c}{3} \cdot \frac{\sin(\alpha)^3}{\alpha - \sin(\alpha) \cdot \cos(\alpha)}$$

Equilibrium in forces $\sum X = 0$

$$P_n = C - \sum_{i=1}^{n_s} T_i = 0.85 \cdot f'_c \cdot A_c - \sum_{i=1}^{n_s} (A_{s,i} \cdot f_{s,i})$$

Equilibrium in moments $\sum M = 0$

$$M_n = P_n \cdot e = C \cdot x_c + \sum_{i=1}^{n_s} \left[T_i \cdot \left(d_i - \frac{D_c}{2} \right) \right]$$

$$M_n = P_n \cdot e = 0.85 \cdot f'_c \cdot A_c \cdot x_c + \sum_{i=1}^{n_s} [A_{s,i} \cdot f_{s,i} \cdot (d_i - r_c)]$$

Conditions of strain compatibility

$$\frac{\epsilon_{s,i}}{\epsilon_u} = \frac{d_i - c}{c}$$

$$f_{s,i} = E_s \cdot \epsilon_{s,i} = E_s \cdot \epsilon_u \cdot \frac{d_i - c}{c} \quad \text{with} \quad |f_{s,i}| \leq f_y$$

Example 14.5

Required strength	$P_u := 3437.31 \text{ kN}$
	$M_u := 42.53 \text{ kN}\cdot\text{m}$
Materials	$f'_c := 20 \text{ MPa}$
	$f_y := 390 \text{ MPa}$

Solution

Determination of concrete dimension

$$\phi := 0.70$$

$$\text{Assume } \rho_g := 0.02$$

$$A_g := \frac{\frac{P_u}{0.85 \cdot \phi}}{0.85 \cdot f'_c \cdot (1 - \rho_g) + f_y \cdot \rho_g} = 2361.812 \cdot \text{cm}^2$$

$$D_c := \text{Ceil} \left(\sqrt{\frac{A_g}{\frac{\pi}{4}}}, 50 \text{ mm} \right) = 550 \cdot \text{mm}$$

$$A_g := \frac{\pi \cdot D_c^2}{4} = 2375.829 \cdot \text{cm}^2$$

Determination of steel area

$$A_{st} := \frac{\frac{P_u}{0.85 \cdot \phi} - 0.85 \cdot f'_c \cdot A_g}{-0.85 \cdot f'_c + f_y} = 46.597 \cdot \text{cm}^2$$

$$D_s := D_c - \left(30 \text{ mm} + 10 \text{ mm} + \frac{20 \text{ mm}}{2} \right) \cdot 2 = 450 \cdot \text{mm}$$

$$n_s := \text{ceil} \left(\frac{\pi \cdot D_s}{100 \text{ mm}} \right) = 15$$

$$A_{s0} := \frac{\pi \cdot (20 \text{ mm})^2}{4} = 3.142 \cdot \text{cm}^2$$

$$A_{st} := n_s \cdot A_{s0} = 47.124 \cdot \text{cm}^2$$

$$s := \frac{\pi \cdot D_s}{n_s} = 94.248 \cdot \text{mm}$$

Interaction diagram for column strength

$$\phi P_{n,\max} := 0.85 \cdot \phi \cdot [0.85 \cdot f'_c \cdot (A_g - A_{st}) + f_y \cdot A_{st}] = 3448.996 \cdot \text{kN}$$

$$\beta_1 := 0.65 \max \left[\left(0.85 - 0.05 \cdot \frac{f'_c - 27.6 \text{MPa}}{6.9 \text{MPa}} \right) \min 0.85 \right] = 0.85$$

$$E_s := 2 \cdot 10^5 \text{MPa} \quad \varepsilon_u := 0.003$$

$$c(a) := \frac{a}{\beta_1}$$

$$i := 1 \dots n_s \quad \alpha_{s_i} := \frac{2 \cdot \pi}{n_s} \cdot (i - 1) \quad d_i := \frac{D_c}{2} - \frac{D_s}{2} \cdot \cos(\alpha_{s_i})$$

$$d_t := \max(d) = 495.083 \cdot \text{mm}$$

$$\phi(a) := \begin{cases} \varepsilon_t \leftarrow \varepsilon_u \cdot \frac{d_t - c(a)}{c(a)} \\ \phi \leftarrow 0.70 \max \left[\left(\frac{1.7 + 200 \cdot \varepsilon_t}{3} \right) \min 0.9 \right] \end{cases}$$

$$f_s(i, a) := \begin{cases} \varepsilon_s \leftarrow \varepsilon_u \cdot \frac{d_i - c(a)}{c(a)} \\ \text{sign}(\varepsilon_s) \cdot \min(|E_s \cdot \varepsilon_s|, f_y) \end{cases}$$

$$r_c := \frac{D_c}{2}$$

$$\alpha(a) := \arccos \left(\frac{r_c - a}{r_c} \right)$$

$$x_c(a) := \frac{2 \cdot r_c}{3} \cdot \frac{\sin(\alpha(a))^3}{\alpha(a) - \sin(\alpha(a)) \cdot \cos(\alpha(a))}$$

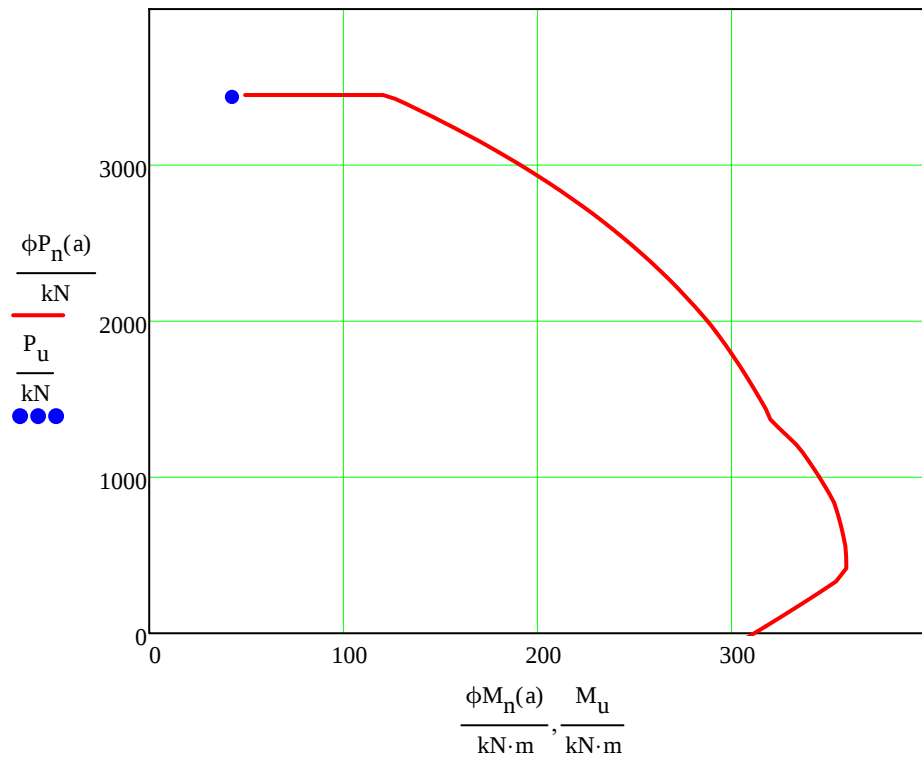
$$A_c(a) := r_c^2 \cdot (\alpha(a) - \sin(\alpha(a)) \cdot \cos(\alpha(a)))$$

$$\phi P_n(a) := \min \left[\phi(a) \cdot \left[0.85 \cdot f'_c \cdot A_c(a) - \sum_{i=1}^{n_s} (A_{s0} \cdot f_s(i, a)) \right], \phi P_{n.\max} \right]$$

$$\phi M_n(a) := \phi(a) \cdot \left[0.85 \cdot f'_c \cdot A_c(a) \cdot x_c(a) + \sum_{i=1}^{n_s} [A_{s0} \cdot f_s(i, a) \cdot (d_i - r_c)] \right]$$

$$a := 0, \frac{D_c}{100} \dots D_c$$

Interaction diagram for column strength



3. Long (Slender) Columns

Stability index

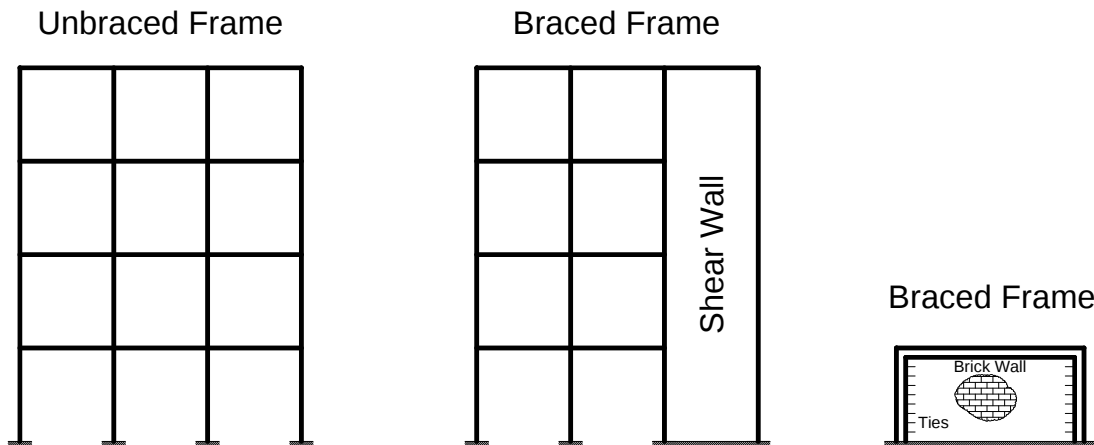
$$Q = \frac{\Sigma P_u \cdot \Delta_0}{V_u \cdot L_c}$$

where

$\Sigma P_u, V_u$ = total vertical force and story shear
 Δ_0 = relative deflection between column ends
 L_c = center-to-center length of column

$Q \leq 0.05$: Frame is nonsway (braced)

$Q > 0.05$: Frame is sway (unbraced)



Slenderness of column

The column is short, if

In nonsway frame:

$$\frac{k \cdot L_u}{r} \leq \min \left(34 - 12 \cdot \frac{M_1}{M_2}, 40 \right)$$

In sway frame:

$$\frac{k \cdot L_u}{r} \leq 22$$

where

$$M_1 = \min(M_A, M_B) \quad M_2 = \max(M_A, M_B)$$

= minimum and maximum moments at the ends of column

L_u = unsupported length of column

r = radius of gyration

$$r = \sqrt{\frac{I}{A}}$$

I, A = moment of inertia and area of column section

k = effective length factor

$$k = k(\psi_A, \psi_B)$$

ψ_A, ψ_B = degree of end restraint (release)

$$\psi = \frac{\sum \left(\frac{EI_c}{L_c} \right)}{\sum \left(\frac{EI_b}{L_b} \right)}$$

$\psi = 0$: column is fixed

$\psi = \infty$: column is pinned

Moments of inertia

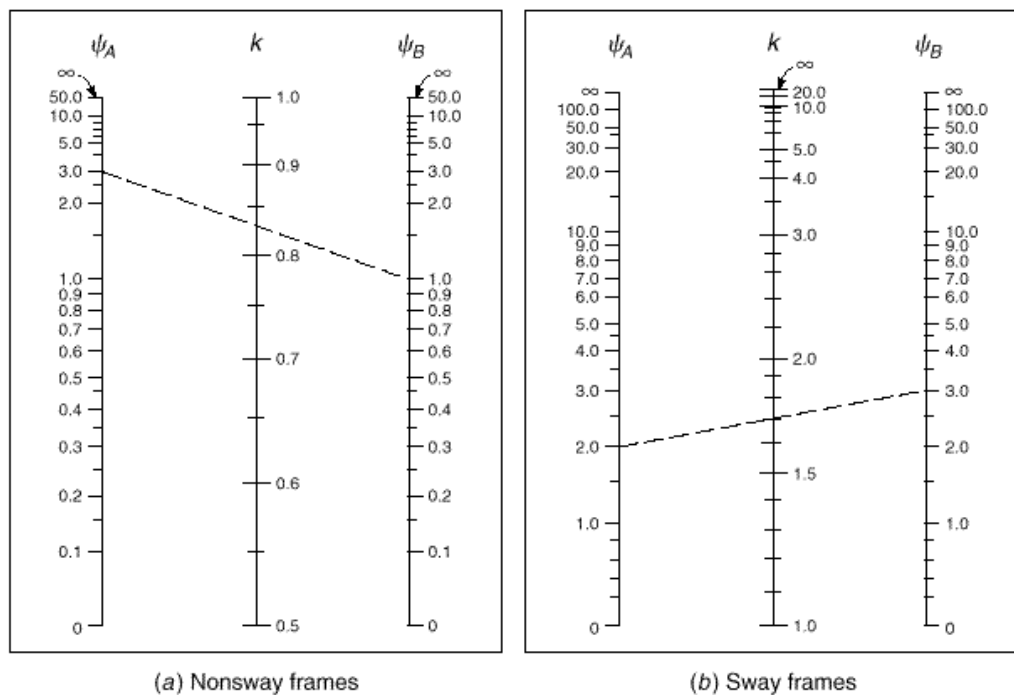
For column $I_c = 0.70I_g$

For beam $I_b = 0.35I_g$

I_g = moment of inertia of gross section

Determination of effective length factor

Way 1. Using graph



Way 2. Using equations

For braced frames:

$$\frac{\psi_A \cdot \psi_B}{4} \cdot \left(\frac{\pi}{k} \right)^2 + \frac{\psi_A + \psi_B}{2} \cdot \left(1 - \frac{\frac{\pi}{k}}{\tan\left(\frac{\pi}{k}\right)} \right) + \frac{2 \cdot \tan\left(\frac{\pi}{2 \cdot k}\right)}{\frac{\pi}{k}} = 1$$

For unbraced frames:

$$\frac{\psi_A \cdot \psi_B \cdot \left(\frac{\pi}{k}\right)^2 - 36}{6 \cdot (\psi_A + \psi_B)} = \frac{\frac{\pi}{k}}{\tan\left(\frac{\pi}{k}\right)}$$

Way 3. Using approximate relations

In nonsway frames:

$$k = 0.7 + 0.05 \cdot (\psi_A + \psi_B) \leq 1.0$$

$$k = 0.85 + 0.05 \cdot \psi_{\min} \leq 1.0$$

$$\psi_{\min} = \min(\psi_A, \psi_B)$$

In sway frames:

Case $\psi_m < 2$

$$k = \frac{20 - \psi_m}{20} \cdot \sqrt{1 + \psi_m}$$

Case $\psi_m \geq 2$

$$k = 0.9 \cdot \sqrt{1 + \psi_m}$$

$$\psi_m = \frac{\psi_A + \psi_B}{2}$$

Case of column is hinged at one end

$$k = 2.0 + 0.3 \cdot \psi$$

ψ is the value in the restrained end.

Moment on column

$$M_c = M_2 \cdot \delta_{ns} \geq M_{2,\min} \cdot \delta_{ns}$$

where

$$M_{2,\min} = P_u \cdot (15\text{mm} + 0.03h)$$

Moment magnification factor

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75 \cdot P_c}} \geq 1$$

Euler's critical load

$$P_c = \frac{\pi^2 \cdot EI}{(k \cdot L_u)^2}$$

$$EI = \frac{0.4 \cdot E_c \cdot I_g}{1 + \beta_d}$$

$$\beta_d = \frac{1.2 \cdot P_D}{1.2 \cdot P_D + 1.6 \cdot P_L}$$

Coefficient

$$C_m = 0.6 + 0.4 \cdot \frac{M_1}{M_2} \geq 0.4$$

Example 14.6

Required strength

$$P_u := 6402.35 \text{ kN}$$

$$\begin{pmatrix} P_D \\ P_L \end{pmatrix} := \begin{pmatrix} 4273.41 \text{ kN} \\ 796.25 \text{ kN} \end{pmatrix}$$

$$M_A := 77.75 \text{ kN} \cdot \text{m}$$

$$M_B := -122.68 \text{ kN} \cdot \text{m}$$

Length of column

$$L_c := 7.8 \text{ m}$$

Upper and lower columns

$$\begin{pmatrix} b_a \\ h_a \\ L_a \end{pmatrix} := \begin{pmatrix} 60 \text{ cm} \\ 60 \text{ cm} \\ 3.6 \text{ m} \end{pmatrix}$$

$$\begin{pmatrix} b_b \\ h_b \\ L_b \end{pmatrix} := \begin{pmatrix} 65 \text{ cm} \\ 65 \text{ cm} \\ 1.5 \text{ m} \end{pmatrix}$$

Upper and lower beams

$$\begin{pmatrix} b_{a1} \\ h_{a1} \\ L_{a1} \end{pmatrix} := \begin{pmatrix} 30 \text{ cm} \\ 50 \text{ cm} \\ 6 \text{ m} \end{pmatrix}$$

$$\begin{pmatrix} b_{a2} \\ h_{a2} \\ L_{a2} \end{pmatrix} := \begin{pmatrix} 30 \text{ cm} \\ 50 \text{ cm} \\ 6 \text{ m} \end{pmatrix}$$

$$\begin{pmatrix} b_{b1} \\ h_{b1} \\ L_{b1} \end{pmatrix} := \begin{pmatrix} 30 \text{ cm} \\ 50 \text{ cm} \\ 6 \text{ m} \end{pmatrix}$$

$$\begin{pmatrix} b_{b2} \\ h_{b2} \\ L_{b2} \end{pmatrix} := \begin{pmatrix} 30 \text{ cm} \\ 50 \text{ cm} \\ 6 \text{ m} \end{pmatrix}$$

Materials

$$f_c := 30 \text{ MPa}$$

$$f_y := 390 \text{ MPa}$$

Solution

Determination of concrete dimension

$$\phi := 0.65$$

Assume $\rho_g := 0.03$

$$A_g := \frac{\frac{P_u}{0.80 \cdot \phi}}{0.85 \cdot f'_c \cdot (1 - \rho_g) + f_y \cdot \rho_g} = 3379.226 \cdot \text{cm}^2$$

Proportion of column section $k = \frac{b}{h} \quad k := \frac{60}{60}$

$$h := \sqrt{\frac{A_g}{k}} = 581.311 \cdot \text{mm} \quad b := k \cdot h = 581.311 \cdot \text{mm}$$

$$h := \text{Ceil}(h, 50\text{mm}) = 600 \cdot \text{mm}$$

$$b := \text{Ceil}(b, 50\text{mm}) = 600 \cdot \text{mm}$$

$$\begin{pmatrix} b \\ h \end{pmatrix} = \begin{pmatrix} 600 \\ 600 \end{pmatrix} \cdot \text{mm}$$

Determination of steel area

$$A_g := b \cdot h = 3.6 \times 10^3 \cdot \text{cm}^2$$

$$A_{st} := \frac{\frac{P_u}{0.80 \cdot \phi} - 0.85 \cdot f'_c \cdot A_g}{-0.85 \cdot f'_c + f_y} = 85.932 \cdot \text{cm}^2$$

$$20 \cdot \frac{\pi \cdot (25\text{mm})^2}{4} = 98.175 \cdot \text{cm}^2$$

$$\text{Bars} := \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \cdot 25\text{mm}$$

$$A_{s0} := \frac{\pi \cdot \text{Bars}^2}{4}$$

$$i := 1 \dots \text{cols}(A_{s0})$$

$$A_{s1_i} := \sum A_{s0}^{(i)}$$

$$A_s := A_{s1}$$

$$A_{st} := \sum A_s$$

$$A_{st} = 98.175 \cdot \text{cm}^2$$

$$\frac{A_{st}}{A_g} = 0.027$$

$$n_s := \text{rows}(A_s)$$

$$n_s = 6$$

$$\text{Cover} := 40\text{mm} + 10\text{mm} + \frac{25\text{mm}}{2} = 62.5 \cdot \text{mm}$$

$$\begin{aligned}
 d1_1 &:= \text{Cover} & \Delta s &:= \frac{h - \text{Cover} \cdot 2}{n_s - 1} = 95 \cdot \text{mm} \\
 i &:= 2 \dots n_s & d1_i &:= d1_{i-1} + \Delta s \\
 d &:= d1 & d &= \begin{pmatrix} 62.5 \\ 157.5 \\ 252.5 \\ 347.5 \\ 442.5 \\ 537.5 \end{pmatrix} \cdot \text{mm}
 \end{aligned}$$

$$\phi P_{n,\max} := 0.80 \cdot \phi \cdot \left[0.85 \cdot f_c \cdot (A_g - A_{st}) + f_y \cdot A_{st} \right] = 6634.405 \cdot \text{kN}$$

Slenderness of column

$$\text{Stability index} \quad Q := 0$$

$$\text{Radius of gyration} \quad r := \frac{h}{\sqrt{12}} = 0.173 \text{ m}$$

Modulus of elasticity

$$w_c := 24 \frac{\text{kN}}{\text{m}^3}$$

$$E_c := 44 \text{ MPa} \cdot \left(\frac{w_c}{\frac{\text{kN}}{\text{m}^3}} \right)^{1.5} \cdot \sqrt{\frac{f_c}{\text{MPa}}} = 2.834 \times 10^4 \cdot \text{MPa}$$

Degree of end restraint

$$I_{a1} := 0.35 \cdot \frac{b_{a1} \cdot h_{a1}^3}{12}$$

$$I_{a2} := 0.35 \cdot \frac{b_{a2} \cdot h_{a2}^3}{12}$$

$$I_{b1} := 0.35 \cdot \frac{b_{b1} \cdot h_{b1}^3}{12}$$

$$I_{b2} := 0.35 \cdot \frac{b_{b2} \cdot h_{b2}^3}{12}$$

$$I_{ca} := 0.70 \cdot \frac{b_a \cdot h_a^3}{12}$$

$$I_{cb} := 0.70 \cdot \frac{b_b \cdot h_b^3}{12}$$

$$I_c := 0.70 \cdot \frac{b \cdot h^3}{12}$$

$$\Sigma i_{ca} := \frac{E_c \cdot I_{ca}}{L_a} + \frac{E_c \cdot I_c}{L_c}$$

$$\Sigma i_{cb} := \frac{E_c \cdot I_{cb}}{L_b} + \frac{E_c \cdot I_c}{L_c}$$

$$\Sigma i_{ba} := \frac{E_c \cdot I_{a1}}{L_{a1}} + \frac{E_c \cdot I_{a2}}{L_{a2}}$$

$$\Sigma i_{bb} := \frac{E_c \cdot I_{b1}}{L_{b1}} + \frac{E_c \cdot I_{b2}}{L_{b2}}$$

$$\psi_A := \frac{\Sigma i_{ca}}{\Sigma i_{ba}} = 8.418$$

$$\psi_B := \frac{\Sigma i_{cb}}{\Sigma i_{bb}} = 21.699$$

Effective length factor

$$k := 0.6$$

Given

$$\frac{\psi_A \cdot \psi_B}{4} \cdot \left(\frac{\pi}{k}\right)^2 + \frac{\psi_A + \psi_B}{2} \cdot \left(1 - \frac{\frac{\pi}{k}}{\tan\left(\frac{\pi}{k}\right)}\right) + \frac{2 \cdot \tan\left(\frac{\pi}{2 \cdot k}\right)}{\frac{\pi}{k}} = 1$$

$$k \geq 0.5 \quad k \leq 1.0$$

$$k := \text{Find}(k)$$

$$k = 0.969$$

Checking for long column

$$M_1 := \begin{cases} M_A & \text{if } |M_A| < |M_B| \\ M_B & \text{otherwise} \end{cases} \quad M_2 := \begin{cases} M_B & \text{if } |M_A| < |M_B| \\ M_A & \text{otherwise} \end{cases}$$

$$M_1 = 77.75 \cdot \text{kN} \cdot \text{m}$$

$$M_2 = -122.68 \cdot \text{kN} \cdot \text{m}$$

$$L_u := L_c - \frac{\max(h_{a1}, h_{a2}) + \max(h_{b1}, h_{b2})}{2} = 7.3 \text{ m}$$

$$\frac{k \cdot L_u}{r} = 40.834$$

$$\min\left(34 - 12 \cdot \frac{M_1}{M_2}, 40\right) = 40$$

$$\text{The_column} := \begin{cases} \text{"is short"} & \text{if } \frac{k \cdot L_u}{r} \leq \min\left(34 - 12 \cdot \frac{M_1}{M_2}, 40\right) \\ \text{"is long"} & \text{otherwise} \end{cases}$$

$$\text{The_column} = \text{"is long"}$$

Case of long column

$$\beta_d := \frac{1.2 \cdot P_D}{1.2 \cdot P_D + 1.6 \cdot P_L} = 0.801$$

$$I_g := \frac{b \cdot h^3}{12} \quad EI := \frac{0.4 \cdot E_c \cdot I_g}{1 + \beta_d}$$

$$P_c := \frac{\pi^2 \cdot EI}{(k \cdot L_u)^2} = 13409.955 \cdot \text{kN}$$

$$C_m := \max\left(0.6 + 0.4 \cdot \frac{M_1}{M_2}, 0.4\right) = 0.4$$

$$\delta_{ns} := \max \left(\frac{C_m}{1 - \frac{P_u}{0.75 \cdot P_c}}, 1 \right) = 1.101$$

$$M_{2,min} := P_u \cdot (15mm + 0.03 \cdot h) = 211.278 \cdot kN \cdot m$$

$$M_c := \begin{cases} \delta_{ns} \cdot \max(|M_2|, M_{2,min}) & \text{if The_column} = \text{"is long"} \\ \max(|M_2|, M_{2,min}) & \text{otherwise} \end{cases}$$

Interaction diagram for column strength

$$c(a) := \frac{a}{\beta_1}$$

$$d_t := \max(d) = 537.5 \cdot mm$$

$$\phi(a) := \begin{cases} \epsilon_t \leftarrow \epsilon_u \cdot \frac{d_t - c(a)}{c(a)} \\ \phi \leftarrow 0.65 \max \left[\left(\frac{1.45 + 250 \cdot \epsilon_t}{3} \right) \min 0.90 \right] \end{cases}$$

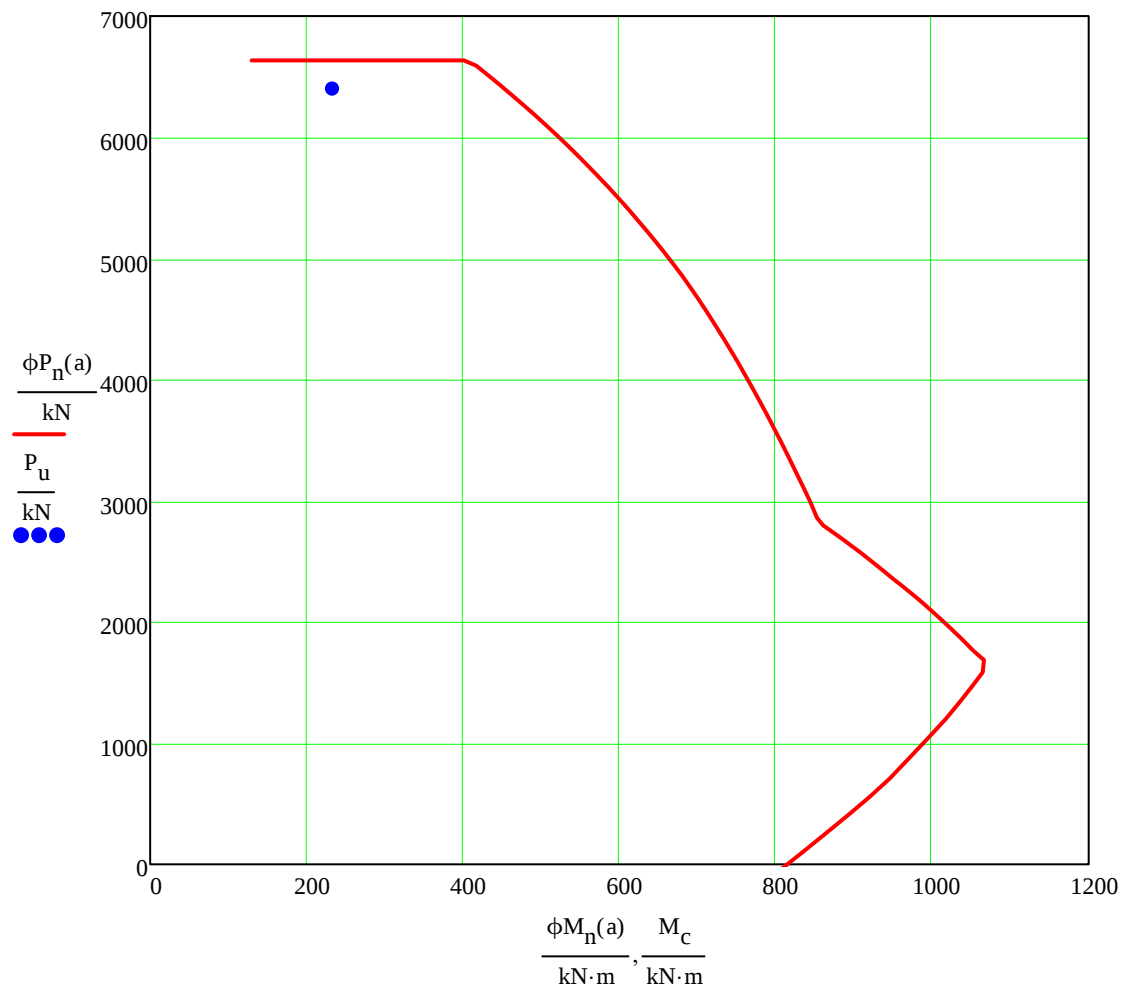
$$f_s(i, a) := \begin{cases} \epsilon_s \leftarrow \epsilon_u \cdot \frac{d_i - c(a)}{c(a)} \\ \text{sign}(\epsilon_s) \cdot \min(E_s \cdot |\epsilon_s|, f_y) \end{cases}$$

$$\phi P_n(a) := \min \left[\phi(a) \cdot \left[0.85 \cdot f_c \cdot a \cdot b - \sum_{i=1}^{n_s} (A_{s_i} \cdot f_s(i, a)) \right], \phi P_{n,max} \right]$$

$$\phi M_n(a) := \phi(a) \cdot \left[0.85 \cdot f_c \cdot a \cdot b \cdot \left(\frac{h}{2} - \frac{a}{2} \right) + \sum_{i=1}^{n_s} \left[A_{s_i} \cdot f_s(i, a) \cdot \left(d_i - \frac{h}{2} \right) \right] \right]$$

$$a := 0, \frac{h}{100} \dots h$$

Interaction diagram for column strength



15. Footing Design

A. Determination of Footing Dimension

Required area of footing

$$A_{\text{req}} = \frac{P_D + P_L}{q_e}$$

where

P_D, P_L = dead and live loads on footing

q_e = effective bearing capacity of soil

$$q_e = q_a - 20 \frac{\text{kN}}{\text{m}^3} \cdot H$$

q_a = allowable bearing capacity of soil with FS = 2.5..3

$20 \frac{\text{kN}}{\text{m}^3}$ = average density of soil and concrete

H = depth of foundation

Checking for maximum stress of soil under footing

$$q_{\text{max}} \leq q_u$$

$$q_{\text{max}} = \begin{cases} \frac{P}{B \cdot L} \cdot \left(1 + \frac{6 \cdot e}{L} \right) & \text{if } e \leq \frac{L}{6} \\ \frac{4P}{3 \cdot B \cdot (L - 2 \cdot e)} & \text{if } e > \frac{L}{6} \end{cases}$$

where

q_u = design bearing capacity of soil

$$q_u = q_a \cdot \frac{1.2P_D + 1.6 \cdot P_L}{P_D + P_L}$$

P = axial load on footing

$$P = 1.2 \cdot (P_D + P_0) + 1.6 \cdot P_L$$

$$P_0 = 20 \frac{\text{kN}}{\text{m}^3} \cdot H \cdot B \cdot L$$

e = eccentricity of load

$$e = \frac{M}{P}$$

L, B = long and width of footing

B. Determination of Depth of Footing

Checking for Punching

$$V_u \leq \phi V_c$$

where

V_u = punching shear

V_c = punching shear strength

$\phi = 0.75$ is a strength reduction factor for shear

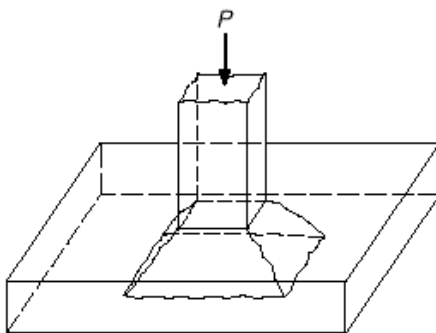


FIGURE 16.6
Punching-shear failure in single footing.

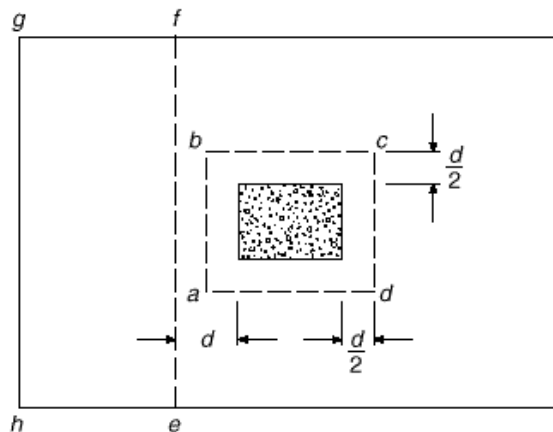


FIGURE 16.7
Critical sections for shear.

Punching shear

$$V_u = q_u \cdot (A - A_0)$$

$$A = B \cdot L$$

$$A_0 = (b_c + d) \cdot (h_c + d)$$

Punching shear strength

$$V_c = 4 \cdot \sqrt{f'_c} \cdot b_0 \cdot d \quad (\text{in psi})$$

$$V_c = 0.332 \cdot \sqrt{f'_c} \cdot b_0 \cdot d \quad (\text{in MPa})$$

$$b_0 = [(b_c + d) + (h_c + d)] \cdot 2$$

Checking for Beam Shear

$$V_{u1} \leq \phi V_{c1}$$

$$V_{u2} \leq \phi V_{c2}$$

where

V_{u1}, V_{u2} = beam shears

V_{c1}, V_{c2} = beam shear strength

Beam shears

$$V_{u1} = q_u \cdot B \cdot \left(\frac{L}{2} - \frac{h_c}{2} - d \right)$$

$$V_{u2} = q_u \cdot L \cdot \left(\frac{B}{2} - \frac{b_c}{2} - d \right)$$

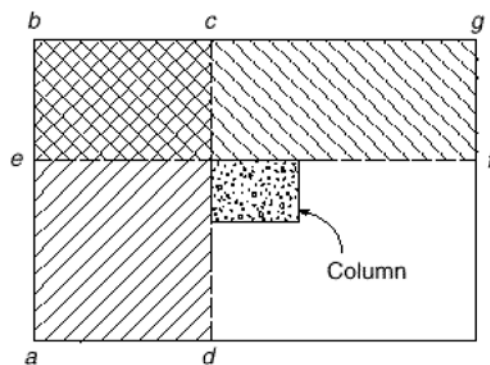
Beam shear strength

$$V_{c1} = 0.166 \cdot \sqrt{f'_c} \cdot B \cdot d$$

$$V_{c2} = 0.166 \cdot \sqrt{f'_c} \cdot L \cdot d$$

C. Determination of Steel Area

FIGURE 16.9
Critical sections for bending
and bond.



Steel re-bars in long direction

Required strength

$$q_1 = q_u \cdot B \quad L_1 = \frac{L}{2} - \frac{h_c}{2}$$
$$M_{u1} = \frac{q_1 \cdot L_1^2}{2}$$

Design section: rectangular singly reinforced beam of $B \times d$

Steel re-bars in short direction

Required strength

$$q_2 = q_u \cdot L \quad L_2 = \frac{B}{2} - \frac{b_c}{2}$$
$$M_{u2} = \frac{q_2 \cdot L_2^2}{2}$$

Design section: rectangular singly reinforced beam of $L \times d$

Example 15.1

Required strength

$$P_D := 484.71 \text{ kN}$$

$$P_L := 228.56 \text{ kN}$$

$$M_u := 5.03 \text{ kN} \cdot \text{m}$$

$$\frac{P_L}{P_D + P_L} = 0.32$$

Dimension of column stub

$$b_c := 350 \text{ mm}$$

$$h_c := 350 \text{ mm}$$

Depth of foundation

$$H := 2.0 \text{ m}$$

Allowable bearing capacity of soil

$$q_a := 178.33 \frac{\text{kN}}{\text{m}^2} \cdot \frac{3}{2.5} = 213.996 \cdot \frac{\text{kN}}{\text{m}^2}$$

Materials

$$f'_c := 25 \text{ MPa}$$

$$f_y := 390 \text{ MPa}$$

Solution

Determination of Dimension of Footing

Effective bearing capacity of soil

$$q_e := q_a - 20 \frac{\text{kN}}{\text{m}^3} \cdot H = 173.996 \cdot \frac{\text{kN}}{\text{m}^2}$$

Required area of footing

$$A_{\text{req}} := \frac{P_D + P_L}{q_e} = 4.099 \text{ m}^2$$

Footing proportion $k = \frac{B}{L}$

$$k := \frac{2}{2.1}$$

$$L := \sqrt{\frac{A_{\text{req}}}{k}} = 2.075 \cdot \text{m}$$

$$B := k \cdot L = 1.976 \text{ m}$$

$$L := \text{Ceil}(L, 50\text{mm}) = 2.1 \text{ m}$$

$$B := \text{Ceil}(B, 50\text{mm}) = 2 \text{ m}$$

$$\begin{pmatrix} B \\ L \end{pmatrix} = \begin{pmatrix} 2 \\ 2.1 \end{pmatrix} \text{ m}$$

Design bearing capacity of soil

$$q_u := q_a \cdot \frac{1.2 \cdot P_D + 1.6 \cdot P_L}{P_D + P_L} = 284.224 \cdot \frac{\text{kN}}{\text{m}^2}$$

Checking for maximum stress of soil

$$P_u := 1.2 \cdot \left(P_D + B \cdot L \cdot H \cdot 20 \frac{\text{kN}}{\text{m}^3} \right) + 1.6 \cdot P_L = 1148.948 \cdot \text{kN}$$

$$e := \frac{M_u}{P_u} = 4.378 \cdot \text{mm}$$

$$q_{\text{max}} := \begin{cases} \frac{P_u}{B \cdot L} \cdot \left(1 + \frac{6 \cdot e}{L} \right) & \text{if } e \leq \frac{L}{6} \\ \frac{4P_u}{3 \cdot B \cdot (L - 2 \cdot e)} & \text{otherwise} \end{cases}$$

$$q_{\text{max}} = 276.981 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$\frac{q_{\text{max}}}{q_u} = 0.975$$

$$\text{Soil} := \begin{cases} \text{"is safe"} & \text{if } q_{\text{max}} \leq q_u \\ \text{"is not safe"} & \text{otherwise} \end{cases}$$

$$\text{Soil} = \text{"is safe"}$$

Determination of depth of footing

Punching shear

$$A_0(d) := (b_c + d) \cdot (h_c + d)$$

$$A := B \cdot L$$

$$V_u(d) := q_u \cdot (A - A_0(d))$$

$$V_u(320\text{mm}) = 1066.154 \cdot \text{kN}$$

Punching shear strength

$$b_0(d) := [(b_c + d) + (h_c + d)] \cdot 2$$

$$\phi := 0.75$$

$$\phi V_c(d) := \phi \cdot 0.332 \text{MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}} \cdot b_0(d) \cdot d$$

$$\phi V_c(320\text{mm}) = 1067.712 \cdot \text{kN}$$

Beam shears

$$V_{u1}(d) := q_u \cdot B \cdot \left(\frac{L}{2} - \frac{h_c}{2} - d \right)$$

$$V_{u1}(300\text{mm}) = 326.858 \cdot \text{kN}$$

$$V_{u2}(d) := q_u \cdot L \cdot \left(\frac{B}{2} - \frac{b_c}{2} - d \right)$$

$$V_{u2}(300\text{mm}) = 313.357 \cdot \text{kN}$$

Beam shear strength

$$\phi V_{c1}(d) := \phi \cdot 0.166 \text{MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}} \cdot B \cdot d$$

$$\phi V_{c1}(300\text{mm}) = 373.5 \cdot \text{kN}$$

$$\phi V_{c2}(d) := \phi \cdot 0.166 \text{MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}} \cdot L \cdot d$$

$$\phi V_{c2}(300\text{mm}) = 392.175 \cdot \text{kN}$$

$$c := 50\text{mm} + 20\text{mm} + \frac{20\text{mm}}{2} = 80 \cdot \text{mm}$$

$$d_{\min} := 150\text{mm} - c = 70 \cdot \text{mm}$$

$$d := \begin{cases} d \leftarrow d_{\min} \\ \text{while } (V_u(d) \geq \phi V_c(d)) \vee (V_{u1}(d) \geq \phi V_{c1}(d)) \vee (V_{u2}(d) \geq \phi V_{c2}(d)) \\ \quad d \leftarrow d + 50\text{mm} \\ d \end{cases}$$

$$d = 320 \cdot \text{mm}$$

$$h := d + c = 400 \cdot \text{mm}$$

Steel reinforcements

$$\rho_{\text{shrinkage}} := \begin{cases} (\text{return } 0.0020) & \text{if } f_y \leq 50\text{ksi} \\ (\text{return } 0.0018) & \text{if } f_y \leq 60\text{ksi} \\ \left(\text{return } \max\left(0.0018 \cdot \frac{60\text{ksi}}{f_y}, 0.0014\right) \right) & \text{otherwise} \end{cases}$$

$$\rho_{\text{shrinkage}} = 0.0018$$

Re-bars in long direction

$$b := B \quad L_n := \frac{L}{2} - \frac{h_c}{2} = 0.875 \text{ m} \quad w_u := q_u \cdot b$$

$$M_u := \frac{w_u \cdot L_n^2}{2} = 217.609 \cdot \text{kN} \cdot \text{m} \quad M_n := \frac{M_u}{0.9} \quad \frac{M_u}{b} = 108.805 \cdot \frac{\text{kN} \cdot \text{m}}{1\text{m}}$$

$$R := \frac{M_n}{b \cdot d^2} = 1.181 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f_c}} \right) = 0.00312$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot h) = 19.944 \cdot \text{cm}^2$$

$$s := 150\text{mm} \quad D := 14\text{mm} \quad n := \text{floor}\left(\frac{b - 75\text{mm} \cdot 2 - D}{s}\right) + 1 = 13$$

$$n \cdot \frac{\pi \cdot D^2}{4} = 20.012 \cdot \text{cm}^2$$

Re-bars in short direction

$$b := L \quad L_n := \frac{B}{2} - \frac{b_c}{2} = 0.825 \text{ m} \quad w_u := q_u \cdot b$$

$$M_u := \frac{w_u \cdot L_n^2}{2} = 203.123 \cdot \text{kN} \cdot \text{m} \quad M_n := \frac{M_u}{0.9} \quad \frac{M_u}{b} = 96.725 \cdot \frac{\text{kN} \cdot \text{m}}{1\text{m}}$$

$$R := \frac{M_n}{b \cdot d^2} = 1.05 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f_c}} \right) = 0.00276$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot h) = 18.554 \cdot \text{cm}^2$$

$$s := 160\text{mm} \quad D := 14\text{mm} \quad n := \text{floor}\left(\frac{b - 75\text{mm} \cdot 2 - D}{s}\right) + 1 = 13$$

$$n \cdot \frac{\pi \cdot D^2}{4} = 20.012 \cdot \text{cm}^2$$

16. Design of Pile Caps

1. Determination of Pile Cap

Number of required piles

$$n = \frac{P_D + P_L}{Q_e}$$

where

P_D, P_L = dead and live loads on pile cap

Q_e = effective bearing capacity of pile

$$Q_e = Q_a - 20 \frac{\text{kN}}{\text{m}^3} \cdot (3 \cdot D)^2 \cdot H$$

$20 \frac{\text{kN}}{\text{m}^3}$ = average density of soil and concrete

D = pile size

H = depth of foundation

Distance between piles = $2 \cdot D \dots 4 \cdot D$

Distance from face of pile to face of pile cap = $\frac{D}{2} \leq 200\text{mm}$

Checking for pile reaction

$$R_i = \frac{P}{n} + \frac{M_y \cdot x_i}{\sum_{k=1}^n (x_k)^2} + \frac{M_x \cdot y_i}{\sum_{k=1}^n (y_k)^2} \leq Q_u$$

where

P = load on pile cap

M_x, M_y = moments on pile cap

Q_u = design bearing capacity of pile

$$Q_u = Q_a \cdot \frac{1.2 \cdot P_D + 1.6 \cdot P_L}{P_D + P_L}$$

2. Depth of Pile Cap

Case of punching

$$V_u \leq \phi \cdot V_c$$

where

$$V_u = \text{punching shear}$$

$$V_u = \sum R_{\text{outside}} = Q_u \cdot \sum n_{\text{outside}}$$

$$V_c = \text{punching shear strength}$$

$$V_c = 0.332 \cdot \sqrt{f_c} \cdot b_0 \cdot d$$

$$b_0 = [(b_c + d) + (h_c + d)] \cdot 2$$

Case of beam shear

$$V_{u1} \leq \phi V_{c1}$$

$$V_{u2} \leq \phi V_{c2}$$

where

$$V_{u1}, V_{u2} = \text{beam shears}$$

$$V_{c1}, V_{c2} = \text{beam shear strength}$$

$$V_{u1} = \max\left(\sum R_{\text{left}}, \sum R_{\text{right}}\right)$$

$$V_{u2} = \max\left(\sum R_{\text{bottom}}, \sum R_{\text{top}}\right)$$

$$V_{c1} = 0.166 \cdot \sqrt{f_c} \cdot B \cdot d$$

$$V_{c2} = 0.166 \cdot \sqrt{f_c} \cdot L \cdot d$$

3. Determination of Steel Reinforcements

In long direction

Required moment:
$$M_{u1} = \max \left[\sum R_{\text{left}} \cdot \left(x_{\text{left}} - \frac{h_c}{2} \right), \sum R_{\text{right}} \cdot \left(x_{\text{right}} - \frac{h_c}{2} \right) \right]$$

Design section: Rectangular singly reinforced of $B \times d$

In short direction

Required moment:
$$M_{u2} = \max \left[\sum R_{\text{bottom}} \cdot \left(x_{\text{bottom}} - \frac{b_c}{2} \right), \sum R_{\text{top}} \cdot \left(x_{\text{top}} - \frac{b_c}{2} \right) \right]$$

Design section: Rectangular singly reinforced of $L \times d$

Example 16.1

Pile size	$D := 300\text{mm}$	$L_p := 9\text{m}$
Allowable bearing capacity of pile	$Q_a := 351.5\text{kN}$	
Loads on pile cap	$P_D := 1769.88\text{kN}$	$P_L := 417.11\text{kN}$
	$M_y := 33.92\text{kN}\cdot\text{m}$	$M_x := 56.82\text{kN}\cdot\text{m}$
Depth of foundation	$H := 1.5\text{m}$	
Column stub	$b_c := 350\text{mm}$	$h_c := 500\text{mm}$
Materials	$f'_c := 25\text{MPa}$	
	$f_y := 390\text{MPa}$	
Diameters of main bar	$\begin{pmatrix} D_1 \\ D_2 \end{pmatrix} := \begin{pmatrix} 16\text{mm} \\ 16\text{mm} \end{pmatrix}$	
Concrete cover	$c := 75\text{mm}$	
Depth of concrete crack	$h_{\text{shrinkage}} := 200\text{mm}$	
Diameter of shrinkage rebar	$D_{\text{shrinkage}} := 12\text{mm}$	



Solution

Design of pile

Required strength of pile concrete

$$A_g := D \cdot D$$

$$f_{c, \text{pile}} := \frac{Q_a}{\frac{1}{4} \cdot A_g} = 15.622 \cdot \text{MPa} \quad \text{Use} \quad f_{c, \text{pile}} := 20 \text{MPa}$$

Steel re-bars

$$A_{st} := 0.005 \cdot A_g = 4.5 \cdot \text{cm}^2 \quad 4 \cdot \frac{\pi \cdot (16 \text{mm})^2}{4} = 8.042 \cdot \text{cm}^2$$

Dimension of pile cap

Effective bearing capacity of pile

$$Q_e := Q_a - 20 \frac{\text{kN}}{\text{m}^3} \cdot (3 \cdot D)^2 \cdot H = 327.2 \cdot \text{kN}$$

Number of piles

$$n := \frac{P_D + P_L}{Q_e} = 6.684$$



Required number of piles

$$\text{ceil}(n) = 7$$

Location of pile

$$X := \begin{pmatrix} -1\text{m} \\ 0 \\ 1\text{m} \\ -0.5\text{m} \\ 0.5\text{m} \\ -1\text{m} \\ 0 \\ 1\text{m} \end{pmatrix}$$

$$Y := \begin{pmatrix} 0.8\text{m} \\ 0.8\text{m} \\ 0.8\text{m} \\ 0 \\ 0 \\ -0.8\text{m} \\ -0.8\text{m} \\ -0.8\text{m} \end{pmatrix}$$



Number of piles

$$n := \text{rows}(X)$$

$$n = 8$$

Dimension of pile cap

$$B := \max(Y) - \min(Y) + \left(\min\left(\frac{D}{2}, 200\text{mm}\right) + \frac{D}{2} \right) \cdot 2 \quad B = 2.2 \text{m}$$

$$L := \max(X) - \min(X) + \left(\min\left(\frac{D}{2}, 200\text{mm}\right) + \frac{D}{2} \right) \cdot 2 \quad L = 2.6 \text{m}$$

Checking for pile reactions

$$Q_u := Q_a \cdot \frac{1.2 \cdot P_D + 1.6 \cdot P_L}{P_D + P_L} = 448.616 \cdot \text{kN}$$

$$P_0 := 20 \frac{\text{kN}}{\text{m}^3} \cdot H \cdot B \cdot L = 171.6 \cdot \text{kN}$$

$$P_u := 1.2 \cdot (P_D + P_0) + 1.6 \cdot P_L = 2997.152 \cdot \text{kN}$$

$$i := 1 \dots n \quad \text{ORIGIN} := 1$$

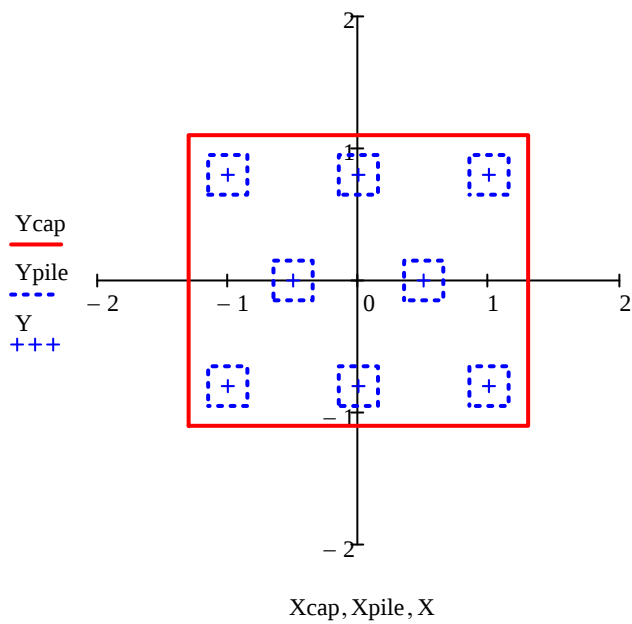
$$R_{u_i} := \frac{P_u}{n} + \frac{M_y \cdot X_i}{\sum_{k=1}^n (X_k)^2} + \frac{M_x \cdot Y_i}{\sum_{k=1}^n (Y_k)^2}$$

$$\frac{R_u}{Q_u} = \begin{pmatrix} 0.845 \\ 0.861 \\ 0.878 \\ 0.827 \\ 0.844 \\ 0.792 \\ 0.809 \\ 0.826 \end{pmatrix}$$

$$X_{\text{cap}} := \begin{pmatrix} -1 \\ 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} \cdot \frac{L}{2} \quad Y_{\text{cap}} := \begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \cdot \frac{B}{2}$$

$$i := 1 \dots n$$

$$X_{\text{pile}}^{(i)} := X_i + \begin{pmatrix} -1 \\ 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} \cdot \frac{D}{2} \quad Y_{\text{pile}}^{(i)} := Y_i + \begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \cdot \frac{D}{2}$$



Determination of Depth of Pile Cap

Punching shear

$$\text{Outside}(d) := \overrightarrow{\left[\left(|X| \geq \frac{h_c}{2} + \frac{d}{2} \right) \vee \left(|Y| \geq \frac{b_c}{2} + \frac{d}{2} \right) \right]}$$

$$V_u(d) := R_u \cdot \text{Outside}(d)$$

$$V_u(700\text{mm}) = 2247.864 \cdot \text{kN}$$

Punching shear strength

$$\phi := 0.75$$

$$b_0(d) := \left[(h_c + d) + (b_c + d) \right] \cdot 2$$

$$\phi V_c(d) := \phi \cdot 0.332 \text{MPa} \cdot \sqrt{\frac{f_c}{\text{MPa}}} \cdot b_0(d) \cdot d$$

$$\phi V_c(700\text{mm}) = 3921.75 \cdot \text{kN}$$

Beam shears

$$\text{Left}(d) := \overrightarrow{\left[X \leq -\left(\frac{h_c}{2} + d \right) \right]}$$

$$\text{Right}(d) := \overrightarrow{\left[X \geq \left(\frac{h_c}{2} + d \right) \right]}$$

$$\text{Bottom}(d) := \overrightarrow{\left[Y \leq -\left(\frac{b_c}{2} + d \right) \right]}$$

$$\text{Top}(d) := \overrightarrow{\left[Y \geq \left(\frac{b_c}{2} + d \right) \right]}$$

$$V_{u1}(d) := \max(R_u \cdot \text{Left}(d), R_u \cdot \text{Right}(d))$$

$$V_{u1}(700\text{mm}) = 764.364 \cdot \text{kN}$$

$$V_{u2}(d) := \max(R_u \cdot \text{Bottom}(d), R_u \cdot \text{Top}(d))$$

$$V_{u2}(700\text{mm}) = 0 \text{ N}$$

Beam shear strength

$$\phi V_{c1}(d) := \phi \cdot 0.166 \text{MPa} \cdot \sqrt{\frac{f_c}{\text{MPa}}} \cdot B \cdot d$$

$$\phi V_{c1}(700\text{mm}) = 958.65 \cdot \text{kN}$$

$$\phi V_{c2}(d) := \phi \cdot 0.166 \text{MPa} \cdot \sqrt{\frac{f_c}{\text{MPa}}} \cdot L \cdot d$$

$$\phi V_{c2}(700\text{mm}) = 1132.95 \cdot \text{kN}$$

Depth of pile cap

$$\text{Cover} := c + D_1 + \frac{D_2}{2} = 99 \cdot \text{mm}$$

$$d := \begin{cases} d \leftarrow 300\text{mm} - \text{Cover} \\ \text{while } (V_u(d) \geq \phi V_c(d)) \vee (V_{u1}(d) \geq \phi V_{c1}(d)) \vee (V_{u2}(d) \geq \phi V_{c2}(d)) \\ \quad d \leftarrow d + 50\text{mm} \\ d \end{cases}$$

$$d = 651 \cdot \text{mm}$$

$$h := d + \text{Cover} = 750 \cdot \text{mm}$$

Steel Reinforcements

$$\rho_{\text{shrinkage}} := \begin{cases} (\text{return } 0.0020) & \text{if } f_y \leq 50\text{ksi} \\ (\text{return } 0.0018) & \text{if } f_y \leq 60\text{ksi} \\ \left(\text{return } \max\left(0.0018 \cdot \frac{60\text{ksi}}{f_y}, 0.0014\right) \right) & \text{otherwise} \end{cases}$$

In long direction

$$b := B$$

$$M_{u1} := \sum \left[\overrightarrow{\text{Left}(0) \cdot R_u \cdot \left(\frac{-h_c}{2} - X \right)} \right] = 643.378 \cdot \text{kN} \cdot \text{m}$$

$$M_{u2} := \sum \left[\overrightarrow{\text{Right}(0) \cdot R_u \cdot \left(X - \frac{h_c}{2} \right)} \right] = 667.876 \cdot \text{kN} \cdot \text{m}$$

$$M_u := \max(M_{u1}, M_{u2}) = 667.876 \cdot \text{kN} \cdot \text{m} \quad M_n := \frac{M_u}{0.9}$$

$$R := \frac{M_n}{b \cdot d^2} = 0.796 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f_c}} \right) = 0.00208$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot h) \quad A_s = 29.797 \cdot \text{cm}^2$$

$$A_{s1} := \frac{\pi \cdot D_1^2}{4} \quad n_1 := \text{ceil}\left(\frac{A_s}{A_{s1}}\right) = 15$$

$$s_1 := \text{Floor}\left[\frac{b - \left(c + \frac{D_1}{2} \right) \cdot 2}{n_1 - 1}, 5\text{mm} \right] = 145 \cdot \text{mm}$$

In short direction

$$b := L$$

$$M_{u1} := \sum \left[\overrightarrow{\text{Bottom}(0) \cdot R_u \cdot \left(\frac{-b_c}{2} - Y \right)} \right] = 680.262 \cdot \text{kN} \cdot \text{m}$$

$$M_{u2} := \sum \left[\overrightarrow{\text{Top}(0) \cdot R_u \cdot \left(Y - \frac{b_c}{2} \right)} \right] = 724.653 \cdot \text{kN} \cdot \text{m}$$

$$M_u := \max(M_{u1}, M_{u2}) = 724.653 \cdot \text{kN} \cdot \text{m} \quad M_n := \frac{M_u}{0.9}$$

$$R := \frac{M_n}{b \cdot d^2} = 0.731 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right) = 0.00191$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot h) \quad A_s = 35.1 \cdot \text{cm}^2$$

$$A_{s2} := \frac{\pi \cdot D_2^2}{4} \quad n_2 := \text{ceil}\left(\frac{A_s}{A_{s2}}\right) = 18$$

$$s_2 := \text{Floor}\left[\frac{b - \left(c + \frac{D_2}{2}\right) \cdot 2}{n_2 - 1}, 5\text{mm}\right] = 140 \cdot \text{mm}$$

Shrinkage reinforcement

$$b := 1\text{m} \quad h_{\text{shrinkage}} := \begin{cases} h & \text{if } h_{\text{shrinkage}} = 0 \\ h_{\text{shrinkage}} & \text{otherwise} \end{cases}$$

$$A_s := \rho_{\text{shrinkage}} \cdot b \cdot h_{\text{shrinkage}} = 3.6 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot D_{\text{shrinkage}}^2}{4} \quad n := \frac{A_s}{A_{s0}}$$

$$s_{\text{shrinkage}} := \text{Floor}\left(\frac{b}{n}, 5\text{mm}\right) = 310 \cdot \text{mm}$$

$$\text{Table} := \begin{bmatrix} \frac{L - \left(c + \frac{D_1}{2}\right) \cdot 2}{m} & \frac{D_1}{\text{mm}} & n_1 & \frac{s_1}{\text{mm}} \\ \frac{B - \left(c + \frac{D_2}{2}\right) \cdot 2}{m} & \frac{D_2}{\text{mm}} & n_2 & \frac{s_2}{\text{mm}} \\ \text{"N/A"} & \frac{D_{\text{shrinkage}}}{\text{mm}} & \text{"N/A"} & \frac{s_{\text{shrinkage}}}{\text{mm}} \end{bmatrix}$$



Dimension of pile cap

B= 2.20 m

L= 2.60 m

Depth of pile cap

h= 750 mm

Direction	Length (mm)	Dia. (mm)	NOS	Spacing (mm)
Long	2.43	16	15	145
Short	2.03	16	18	140
Top	N/A	12	N/A	310

17. Slab Design

A. Design of One-Way Slabs

L_a = length of short side

L_b = length of long side

$\frac{L_a}{L_b} < 0.5$: the slab is one-way

$\frac{L_a}{L_b} \geq 0.5$: the slab is two-way

Thickness of one-way slab

Simply supported	$\frac{L_n}{20}$
One end continuous	$\frac{L_n}{24}$
Both ends continuous	$\frac{L_n}{28}$
Cantilever	$\frac{L_n}{10}$

Analysis of one-way slab

Design scheme: continuous beam

Determination of bending moments: using ACI moment coefficients

Design of one-way slab

Design section: rectangular section of 1m x h

Type section: singly reinforced beam

Example 17.1

Span of slab	$L_n := 2\text{m} - 20\text{cm} = 1.8\text{m}$
Live load	$LL := 12 \frac{\text{kN}}{\text{m}^2}$
Materials	$f'_c := 20\text{MPa}$
	$f_y := 390\text{MPa}$

Solution

Thickness of one-way slab

$$t_{\min} := \frac{L_n}{28} = 64.286 \cdot \text{mm}$$

Use $t := 100\text{mm}$

Loads on slab

$$\text{Cover} := 50\text{mm} \cdot 22 \frac{\text{kN}}{\text{m}^3} = 1.1 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$\text{Slab} := t \cdot 25 \frac{\text{kN}}{\text{m}^3} = 2.5 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$\text{Ceiling} := 0.40 \frac{\text{kN}}{\text{m}^2}$$

$$\text{Mechanical} := 0.20 \frac{\text{kN}}{\text{m}^2}$$

$$\text{Partition} := 1.00 \frac{\text{kN}}{\text{m}^2}$$

$$\text{DL} := \text{Cover} + \text{Slab} + \text{Ceiling} + \text{Mechanical} + \text{Partition} = 5.2 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$w_u := 1.2 \cdot \text{DL} + 1.6 \cdot \text{LL} = 25.44 \cdot \frac{\text{kN}}{\text{m}^2}$$

Bending moments

$$M_{\text{support}} := \frac{1}{11} \cdot w_u \cdot L_n^2 = 7.493 \cdot \frac{\text{kN} \cdot \text{m}}{1\text{m}}$$

$$M_{\text{midspan}} := \frac{1}{16} \cdot w_u \cdot L_n^2 = 5.152 \cdot \frac{\text{kN} \cdot \text{m}}{1\text{m}}$$

Steel reinforcements

$$\beta_1 := 0.65 \max \left[\left(0.85 - 0.05 \cdot \frac{f'_c - 27.6 \text{ MPa}}{6.9 \text{ MPa}} \right) \min 0.85 \right] = 0.85$$

$$\varepsilon_u := 0.003$$

$$\rho_{\max} := 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + 0.005} = 0.014$$

$$\rho_{\min} := \max \left(\frac{0.249 \text{ MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}}}{f_y}, \frac{1.379 \text{ MPa}}{f_y} \right) = 0.00354$$

$$\rho_{\text{shrinkage}} := \begin{cases} (\text{return } 0.0020) & \text{if } f_y \leq 50 \text{ ksi} \\ (\text{return } 0.0018) & \text{if } f_y \leq 60 \text{ ksi} \\ \left(\text{return } \max \left(0.0018 \cdot \frac{60 \text{ ksi}}{f_y}, 0.0014 \right) \right) & \text{otherwise} \end{cases}$$

$$\rho_{\text{shrinkage}} = 0.0018$$

Top rebars

$$b := 1 \text{ m} \quad d := t - \left(20 \text{ mm} + \frac{10 \text{ mm}}{2} \right) = 75 \cdot \text{mm}$$

$$M_u := M_{\text{support}} \cdot b = 7.493 \cdot \text{kN} \cdot \text{m}$$

$$M_n := \frac{M_u}{0.9} = 8.326 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_n}{b \cdot d^2} = 1.48 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right) = 0.004 \quad \rho \leq \rho_{\max} = 1$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t) = 2.982 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10 \text{ mm})^2}{4} \quad n := \frac{A_s}{A_{s0}} \quad s_{\max} := \min(3 \cdot t, 450 \text{ mm})$$

$$s := \min \left(\text{Floor} \left(\frac{b}{n}, 10 \text{ mm} \right), s_{\max} \right) = 260 \cdot \text{mm}$$

Bottom rebars

$$M_u := M_{\text{midspan}} \cdot b = 5.152 \cdot \text{kN} \cdot \text{m}$$

$$M_n := \frac{M_u}{0.9} = 5.724 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_n}{b \cdot d^2} = 1.018 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right) = 0.003$$

$$\rho \leq \rho_{\max} = 1$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t) = 2.019 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4}$$

$$n := \frac{A_s}{A_{s0}}$$

$$s_{\max} := \min(3 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\max}\right) = 300 \cdot \text{mm}$$

Link rebars

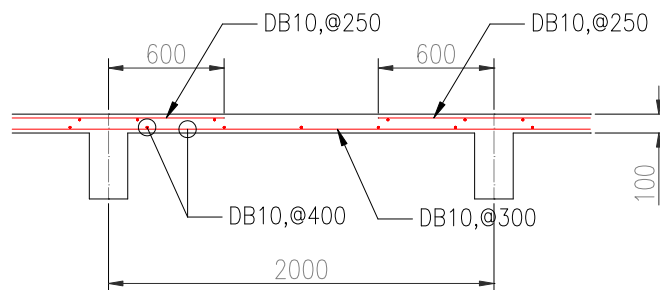
$$A_s := \rho_{\text{shrinkage}} \cdot b \cdot t = 1.8 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4}$$

$$n := \frac{A_s}{A_{s0}}$$

$$s_{\max} := \min(5 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\max}\right) = 430 \cdot \text{mm}$$



B. Design of Two-Way Slabs

Design methods:

- Load distribution method
- Moment coefficient method
- Direct design method (DDM)
- Equivalent frame method
- Strip method
- Yield line method

(1) Load Distribution Method

Principle: Equality of deflection in short and long directions

$$f_a = f_b$$

$$\alpha_a \cdot \frac{w_a \cdot L_a^4}{EI} = \alpha_b \cdot \frac{w_b \cdot L_b^4}{EI}$$

Case $\alpha_a = \alpha_b$

$$\frac{w_a}{w_b} = \frac{L_b^4}{L_a^4} = \frac{1}{\lambda^4} \quad \lambda = \frac{L_a}{L_b}$$

$$w_a + w_b = w_u$$

From which, $w_a = w_u \cdot \frac{1}{1 + \lambda^4}$

$$w_b = w_u \cdot \frac{\lambda^4}{1 + \lambda^4}$$

For $\lambda := 1$	$\frac{1}{1 + \lambda^4} = 0.5$	$\frac{\lambda^4}{1 + \lambda^4} = 0.5$
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For $\lambda := 0.8$	$\frac{1}{1 + \lambda^4} = 0.709$	$\frac{\lambda^4}{1 + \lambda^4} = 0.291$
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For $\lambda := 0.6$	$\frac{1}{1 + \lambda^4} = 0.885$	$\frac{\lambda^4}{1 + \lambda^4} = 0.115$
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For $\lambda := 0.5$	$\frac{1}{1 + \lambda^4} = 0.941$	$\frac{\lambda^4}{1 + \lambda^4} = 0.059$
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For $\lambda := 0.4$	$\frac{1}{1 + \lambda^4} = 0.975$	$\frac{\lambda^4}{1 + \lambda^4} = 0.025$
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Example 17.2

Slab dimension	$L_a := 4.3\text{m}$
	$L_b := 5.5\text{m}$
Live load	$LL := 2.00 \frac{\text{kN}}{\text{m}^2}$
Materials	$f'_c := 20\text{MPa}$
	$f_y := 390\text{MPa}$

Solution

Thickness of two-way slab

$$\text{Perimeter} := (L_a + L_b) \cdot 2$$

$$t_{\min} := \frac{\text{Perimeter}}{180} = 108.889 \cdot \text{mm}$$

$$t := \left(\frac{1}{30} \quad \frac{1}{50} \right) \cdot L_a = (143.333 \quad 86) \cdot \text{mm}$$

Use $t := 120\text{mm}$

Loads on slab

$$\text{SDL} := 50\text{mm} \cdot 22 \frac{\text{kN}}{\text{m}^3} + 0.40 \frac{\text{kN}}{\text{m}^2} + 1.00 \frac{\text{kN}}{\text{m}^2} = 2.5 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$\text{DL} := \text{SDL} + t \cdot 25 \frac{\text{kN}}{\text{m}^3} = 5.5 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$\text{LL} = 2 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$w_u := 1.2 \cdot \text{DL} + 1.6 \cdot \text{LL} = 9.8 \cdot \frac{\text{kN}}{\text{m}^2}$$

Load distribution

$$\lambda := \frac{L_a}{L_b} = 0.782$$

$$w_a := \frac{1}{1 + \lambda^4} \cdot w_u = 7.134 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$w_b := \frac{\lambda^4}{1 + \lambda^4} \cdot w_u = 2.666 \cdot \frac{\text{kN}}{\text{m}^2}$$

Bending moments

$$M_{a,neg} := \frac{1}{11} \cdot w_a \cdot L_a^2 = 11.992 \cdot \frac{\text{kN} \cdot \text{m}}{1\text{m}}$$

$$M_{a,pos} := \frac{1}{16} \cdot w_a \cdot L_a^2 = 8.245 \cdot \frac{\text{kN} \cdot \text{m}}{1\text{m}}$$

$$M_{b,neg} := \frac{1}{11} \cdot w_b \cdot L_b^2 = 7.33 \cdot \frac{\text{kN} \cdot \text{m}}{1\text{m}}$$

$$M_{b,pos} := \frac{1}{16} \cdot w_b \cdot L_b^2 = 5.04 \cdot \frac{\text{kN} \cdot \text{m}}{1\text{m}}$$

Steel reinforcements

$$\beta_1 := 0.65 \max \left[\left(0.85 - 0.05 \cdot \frac{f'_c - 27.6\text{MPa}}{6.9\text{MPa}} \right) \min 0.85 \right] = 0.85$$

$$\epsilon_u := 0.003$$

$$\rho_{\max} := 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + 0.005} = 0.014$$

$$\rho_{\min} := \max \left(\frac{0.249\text{MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}}}{f_y}, \frac{1.379\text{MPa}}{f_y} \right) = 0.00354$$

$$\rho_{\text{shrinkage}} := \begin{cases} \text{return } 0.0020 & \text{if } f_y \leq 50\text{ksi} \\ \text{return } 0.0018 & \text{if } f_y \leq 60\text{ksi} \\ \left(\text{return } \max \left(0.0018 \cdot \frac{60\text{ksi}}{f_y}, 0.0014 \right) \right) & \text{otherwise} \end{cases}$$

$$\rho_{\text{shrinkage}} = 0.0018$$

Top rebar in short direction

$$b := 1\text{m} \quad d := t - \left(20\text{mm} + 10\text{mm} + \frac{10\text{mm}}{2} \right) = 85 \cdot \text{mm}$$

$$M_u := M_{a,neg} \cdot b = 11.992 \cdot \text{kN} \cdot \text{m}$$

$$M_n := \frac{M_u}{0.9} = 13.325 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_n}{b \cdot d^2} = 1.844 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right) = 0.005$$

$$\rho \leq \rho_{\max} = 1$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t) = 4.265 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4} \quad n := \frac{A_s}{A_{s0}}$$

$$s_{\max} := \min(2 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\max}\right) = 180 \cdot \text{mm}$$

Bottom rebar in short direction

$$M_u := M_{a.\text{pos}} \cdot b = 8.245 \cdot \text{kN} \cdot \text{m}$$

$$M_n := \frac{M_u}{0.9} = 9.161 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_n}{b \cdot d^2} = 1.268 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right) = 0.003$$

$$\rho \leq \rho_{\max} = 1$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t) = 2.875 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4} \quad n := \frac{A_s}{A_{s0}}$$

$$s_{\max} := \min(2 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\max}\right) = 240 \cdot \text{mm}$$

Top rebar in long direction

$$M_u := M_{b.\text{neg}} \cdot b = 7.33 \cdot \text{kN} \cdot \text{m}$$

$$M_n := \frac{M_u}{0.9} = 8.145 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_n}{b \cdot d^2} = 1.127 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right) = 0.003$$

$$\rho \leq \rho_{\max} = 1$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t) = 2.544 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4} \quad n := \frac{A_s}{A_{s0}} \quad s_{\max} := \min(2 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\max}\right) = 240 \cdot \text{mm}$$

Bottom rebar in long direction

$$M_u := M_{b,\text{pos}} \cdot b = 5.04 \cdot \text{kN} \cdot \text{m}$$

$$M_n := \frac{M_u}{0.9} = 5.599 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_n}{b \cdot d^2} = 0.775 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}}\right) = 0.002 \quad \rho \leq \rho_{\max} = 1$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t) = 2.16 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4} \quad n := \frac{A_s}{A_{s0}} \quad s_{\max} := \min(2 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\max}\right) = 240 \cdot \text{mm}$$

Shrinkage rebars

$$b := 1\text{m}$$

$$A_s := \rho_{\text{shrinkage}} \cdot b \cdot t = 2.16 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4} \quad n := \frac{A_s}{A_{s0}} \quad s_{\max} := \min(5 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\max}\right) = 360 \cdot \text{mm}$$

(2) Moment Coefficient Method

Negative moments

$$M_{a,neg} = C_{a,neg} \cdot w_u \cdot L_a^2$$

$$M_{b,neg} = C_{b,neg} \cdot w_u \cdot L_b^2$$

Positive moments

$$M_{a,pos} = C_{a,pos,DL} \cdot w_D \cdot L_a^2 + C_{a,pos,LL} \cdot w_L \cdot L_a^2$$

$$M_{b,pos} = C_{b,pos,DL} \cdot w_D \cdot L_b^2 + C_{b,pos,LL} \cdot w_L \cdot L_b^2$$

where $C_{a,neg}, C_{b,neg}, C_{a,pos,DL}, C_{a,pos,LL}, C_{b,pos,DL}, C_{b,pos,LL}$
are tabulated moment coefficients

$$w_D = 1.2 \cdot DL \qquad w_L = 1.6 \cdot LL$$

$$w_u = 1.2 + 1.6 \cdot LL$$

Example 17.3

Slab dimension $L_a := 5.0\text{m} - 25\text{cm} = 4.75\text{ m}$

$$L_b := 5.5\text{m} - 20\text{cm} = 5.3\text{ m}$$

Live load for office $LL := 2.40 \frac{\text{kN}}{\text{m}^2}$

Materials $f'_c := 20\text{MPa} \qquad f_y := 390\text{MPa}$

Boundary conditions in short and long directions

$$\begin{pmatrix} \text{Simple} \\ \text{Continuous} \end{pmatrix} := \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \text{Short} := \begin{pmatrix} \text{Continuous} \\ \text{Continuous} \end{pmatrix}$$

$$\text{Long} := \begin{pmatrix} \text{Continuous} \\ \text{Continuous} \end{pmatrix}$$

Solution

Thickness of two-way slab

$$\text{Perimeter} := (L_a + L_b) \cdot 2$$

$$t_{\min} := \frac{\text{Perimeter}}{180} = 111.667 \cdot \text{mm}$$

$$t := \left(\frac{1}{30} \quad \frac{1}{50} \right) \cdot L_a = (158.333 \quad 95) \cdot \text{mm}$$

$$\text{Use} \quad t := 120 \text{mm}$$

Loads on slab

$$\text{Cover} := 50 \text{mm} \cdot 22 \frac{\text{kN}}{\text{m}^3} = 1.1 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$\text{Slab} := t \cdot 25 \frac{\text{kN}}{\text{m}^3} = 3 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$\text{Ceiling} := 0.40 \frac{\text{kN}}{\text{m}^2}$$

$$\text{Partition} := 1.00 \frac{\text{kN}}{\text{m}^2}$$

$$\text{SDL} := \text{Cover} + \text{Ceiling} + \text{Partition} = 2.5 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$\text{DL} := \text{SDL} + \text{Slab} = 5.5 \cdot \frac{\text{kN}}{\text{m}^2} \quad w_D := 1.2 \cdot \text{DL}$$

$$\text{LL} = 2.4 \cdot \frac{\text{kN}}{\text{m}^2} \quad w_L := 1.6 \cdot \text{LL}$$

$$w_u := 1.2 \cdot \text{DL} + 1.6 \text{LL} = 10.44 \cdot \frac{\text{kN}}{\text{m}^2}$$

Moment coefficients

Table 12.3a									
Coefficients for negative moments in short direction of slab									
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	0.000	0.045	0.000	0.050	0.075	0.071	0.000	0.033	0.061
0.95	0.000	0.050	0.000	0.055	0.079	0.075	0.000	0.038	0.065
0.90	0.000	0.055	0.000	0.060	0.080	0.079	0.000	0.043	0.068
0.85	0.000	0.060	0.000	0.066	0.082	0.083	0.000	0.049	0.072
0.80	0.000	0.065	0.000	0.071	0.083	0.086	0.000	0.055	0.075
0.75	0.000	0.069	0.000	0.076	0.085	0.088	0.000	0.061	0.078
0.70	0.000	0.074	0.000	0.081	0.086	0.091	0.000	0.068	0.081
0.65	0.000	0.077	0.000	0.085	0.087	0.093	0.000	0.074	0.083
0.60	0.000	0.081	0.000	0.089	0.088	0.095	0.000	0.080	0.085
0.55	0.000	0.084	0.000	0.092	0.089	0.096	0.000	0.085	0.086
0.50	0.000	0.086	0.000	0.094	0.090	0.097	0.000	0.089	0.088
Table 12.3b									
Coefficients for negative moments in long direction of slab									
m	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	0.000	0.045	0.076	0.050	0.000	0.000	0.071	0.061	0.033
0.95	0.000	0.041	0.072	0.045	0.000	0.000	0.067	0.056	0.029
0.90	0.000	0.037	0.070	0.040	0.000	0.000	0.062	0.052	0.025
0.85	0.000	0.031	0.065	0.034	0.000	0.000	0.057	0.046	0.021
0.80	0.000	0.027	0.061	0.029	0.000	0.000	0.051	0.041	0.017
0.75	0.000	0.022	0.056	0.024	0.000	0.000	0.044	0.036	0.014
0.70	0.000	0.017	0.050	0.019	0.000	0.000	0.038	0.029	0.011
0.65	0.000	0.014	0.043	0.015	0.000	0.000	0.031	0.024	0.008
0.60	0.000	0.010	0.035	0.011	0.000	0.000	0.024	0.018	0.006
0.55	0.000	0.007	0.028	0.008	0.000	0.000	0.019	0.014	0.005
0.50	0.000	0.006	0.022	0.006	0.000	0.000	0.014	0.010	0.003

ORIGIN := 1

$$\text{Index} := \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \quad I := \text{Index}_{(\text{Short}_1+1, \text{Short}_2+1)} = 3$$

$$J := \text{Index}_{(\text{Long}_1+1, \text{Long}_2+1)} = 3$$

$$\text{Table} := \begin{pmatrix} 1 & 7 & 3 \\ 6 & 4 & 8 \\ 5 & 9 & 2 \end{pmatrix} \quad \text{Case} := \text{Table}_{I,J} = 2$$

$V\lambda := \text{reverse}(V\lambda)$

$\text{Vaneg} := \text{reverse}(\text{Taneg}^{\langle \text{Case} \rangle})$

$\text{Vbneg} := \text{reverse}(\text{Tbneg}^{\langle \text{Case} \rangle})$

$\text{VaposDL} := \text{reverse}(\text{TaposDL}^{\langle \text{Case} \rangle})$

$\text{VbposDL} := \text{reverse}(\text{TbposDL}^{\langle \text{Case} \rangle})$

$\text{VaposLL} := \text{reverse}(\text{TaposLL}^{\langle \text{Case} \rangle})$

$\text{VbposLL} := \text{reverse}(\text{TbposLL}^{\langle \text{Case} \rangle})$

$$\lambda := \frac{L_a}{L_b} = 0.896$$

$\text{vs1} := \text{pspline}(V\lambda, \text{Vaneg})$

$C_{a,\text{neg}} := \text{interp}(\text{vs1}, V\lambda, \text{Vaneg}, \lambda) = 0.055$

$$\begin{aligned}
vs2 &:= \text{pspline}(V\lambda, Vb_{neg}) & C_{b,neg} &:= \text{interp}(vs2, V\lambda, Vb_{neg}, \lambda) = 0.037 \\
vs3 &:= \text{pspline}(V\lambda, Vapos_{DL}) & C_{a,pos,DL} &:= \text{interp}(vs3, V\lambda, Vapos_{DL}, \lambda) = 0.022 \\
vs4 &:= \text{pspline}(V\lambda, Vbpos_{DL}) & C_{b,pos,DL} &:= \text{interp}(vs4, V\lambda, Vbpos_{DL}, \lambda) = 0.014 \\
vs5 &:= \text{pspline}(V\lambda, Vapos_{LL}) & C_{a,pos,LL} &:= \text{interp}(vs5, V\lambda, Vapos_{LL}, \lambda) = 0.034 \\
vs6 &:= \text{pspline}(V\lambda, Vbpos_{LL}) & C_{b,pos,LL} &:= \text{interp}(vs6, V\lambda, Vbpos_{LL}, \lambda) = 0.022
\end{aligned}$$

Bending moments

$$\begin{aligned}
M_{a,neg} &:= C_{a,neg} \cdot w_u \cdot L_a^2 = 13.043 \cdot \frac{\text{kN} \cdot \text{m}}{1\text{m}} \\
M_{b,neg} &:= C_{b,neg} \cdot w_u \cdot L_b^2 = 10.729 \cdot \frac{\text{kN} \cdot \text{m}}{1\text{m}} \\
M_{a,pos} &:= C_{a,pos,DL} \cdot w_D \cdot L_a^2 + C_{a,pos,LL} \cdot w_L \cdot L_a^2 = 6.266 \cdot \frac{\text{kN} \cdot \text{m}}{1\text{m}} \\
M_{b,pos} &:= C_{b,pos,DL} \cdot w_D \cdot L_b^2 + C_{b,pos,LL} \cdot w_L \cdot L_b^2 = 4.911 \cdot \frac{\text{kN} \cdot \text{m}}{1\text{m}}
\end{aligned}$$

Steel reinforcements

$$\beta_1 := 0.65 \max \left[\left(0.85 - 0.05 \cdot \frac{f'_c - 27.6\text{MPa}}{6.9\text{MPa}} \right) \min 0.85 \right] = 0.85$$

$$\epsilon_u := 0.003$$

$$\rho_{\max} := 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + 0.005} = 0.014$$

$$\rho_{\min} := \max \left(\frac{0.249\text{MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}}}{f_y}, \frac{1.379\text{MPa}}{f_y} \right) = 0.00354$$

$$\rho_{\text{shrinkage}} := \begin{cases} \text{return } 0.0020 & \text{if } f_y \leq 50\text{ksi} \\ \text{return } 0.0018 & \text{if } f_y \leq 60\text{ksi} \\ \left(\text{return } \max \left(0.0018 \cdot \frac{60\text{ksi}}{f_y}, 0.0014 \right) \right) & \text{otherwise} \end{cases}$$

$$\rho_{\text{shrinkage}} = 0.0018$$

Top rebars in short direction

$$b := 1\text{m} \quad d := t - \left(20\text{mm} + 10\text{mm} + \frac{10\text{mm}}{2} \right) = 85 \cdot \text{mm}$$

$$M_u := M_{a.neg} \cdot b = 13.043 \cdot \text{kN} \cdot \text{m}$$

$$M_n := \frac{M_u}{0.9} = 14.492 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_n}{b \cdot d^2} = 2.006 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right) = 0.005$$

$$\rho \leq \rho_{\max} = 1$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t) = 4.666 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4} \quad n := \frac{A_s}{A_{s0}}$$

$$s_{\max} := \min(2 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\max}\right) = 160 \cdot \text{mm}$$

Bottom rebars in short direction

$$M_u := M_{a.pos} \cdot b = 6.266 \cdot \text{kN} \cdot \text{m}$$

$$M_n := \frac{M_u}{0.9} = 6.963 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_n}{b \cdot d^2} = 0.964 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right) = 0.003$$

$$\rho \leq \rho_{\max} = 1$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t) = 2.163 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4} \quad n := \frac{A_s}{A_{s0}}$$

$$s_{\max} := \min(2 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\max}\right) = 240 \cdot \text{mm}$$

Top rebars in long direction

$$M_u := M_{b.neg} \cdot b = 10.729 \cdot \text{kN} \cdot \text{m}$$

$$M_n := \frac{M_u}{0.9} = 11.921 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_n}{b \cdot d^2} = 1.65 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right) = 0.004$$

$$\rho \leq \rho_{\max} = 1$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t) = 3.79 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4} \quad n := \frac{A_s}{A_{s0}}$$

$$s_{\max} := \min(2 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\max}\right) = 200 \cdot \text{mm}$$

Bottom rebars in long direction

$$M_u := M_{b,\text{pos}} \cdot b = 4.911 \cdot \text{kN} \cdot \text{m}$$

$$M_n := \frac{M_u}{0.9} = 5.457 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_n}{b \cdot d^2} = 0.755 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right) = 0.002$$

$$\rho \leq \rho_{\max} = 1$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t) = 2.16 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4} \quad n := \frac{A_s}{A_{s0}}$$

$$s_{\max} := \min(2 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\max}\right) = 240 \cdot \text{mm}$$

Shrinkage rebars

$$b := 1\text{m}$$

$$A_s := \rho_{\text{shrinkage}} \cdot b \cdot t = 2.16 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4} \quad n := \frac{A_s}{A_{s0}}$$

$$s_{\max} := \min(5 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\max}\right) = 360 \cdot \text{mm}$$

(3) Direct Design Method (DDM)

Total static moment

$$M_0 = \frac{w_u \cdot L_2 \cdot L_n^2}{8}$$

Longitudinal distribution of moments

$$M_{neg} = C_{neg} \cdot M_0$$

$$M_{pos} = C_{pos} \cdot M_0$$

	(1)	(2)	(3)	(4)	(5)
	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior negative factored moment	0.75	0.70	0.70	0.70	0.65
Positive factored moment	0.63	0.57	0.52	0.50	0.35
Exterior negative factored moment	0	0.16	0.26	0.30	0.65

Lateral distribution of moments

$$M_{neg.col} = C_{neg.col} \cdot M_{neg}$$

$$M_{neg.mid} = C_{neg.mid} \cdot M_{neg}$$

$$M_{pos.col} = C_{pos.col} \cdot M_{pos}$$

$$M_{pos.mid} = C_{pos.mid} \cdot M_{pos}$$

13.6.4.1 — Column strips shall be proportioned to resist the following portions in percent of interior negative factored moments:

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_1 \ell_2/\ell_1) = 0$	75	75	75
$(\alpha_1 \ell_2/\ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

13.6.4.2 — Column strips shall be proportioned to resist the following portions in percent of exterior negative factored moments:

ℓ_2/ℓ_1		0.5	1.0	2.0
$(\alpha_1 \ell_2/\ell_1) = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	75	75	75
$(\alpha_1 \ell_2/\ell_1) \geq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	90	75	45

Linear interpolations shall be made between values shown, where β_t is calculated in Eq. (13-5) and C is calculated in Eq. (13-6).

13.6.4.4 — Column strips shall be proportioned to resist the following portions in percent of positive factored moments:

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_1 \ell_2/\ell_1) = 0$	60	60	60
$(\alpha_1 \ell_2/\ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

13.6.5.1 — Beams between supports shall be proportioned to resist 85 percent of column strip moments if $\alpha_1 \ell_2/\ell_1$ is equal to or greater than 1.0.

13.6.5.2 — For values of $\alpha_1 \ell_2/\ell_1$ between 1.0 and zero, proportion of column strip moments resisted by beams shall be obtained by linear interpolation between 85 and zero percent.

Example 17.4

Slab dimension	$L_a := 4\text{m}$	$L_b := 6\text{m}$
Live load for hospital	$LL := 3.00 \frac{\text{kN}}{\text{m}^2}$	
Materials	$f'_c := 25\text{MPa}$	$f_y := 390\text{MPa}$

Solution

Section of beam in long direction

$$L := L_b = 6 \text{ m}$$

$$h := \left(\frac{1}{10} \quad \frac{1}{15} \right) \cdot L = (600 \quad 400) \cdot \text{mm} \quad h := 500 \text{ mm}$$

$$b := (0.3 \quad 0.6) \cdot h = (150 \quad 300) \cdot \text{mm} \quad b := 250 \text{ mm}$$

$$\begin{pmatrix} b_b \\ h_b \end{pmatrix} := \begin{pmatrix} b \\ h \end{pmatrix}$$

Section of beam in short direction

$$L := L_a = 4 \text{ m}$$

$$h := \left(\frac{1}{10} \quad \frac{1}{15} \right) \cdot L = (400 \quad 266.667) \cdot \text{mm} \quad h := 300 \text{ mm}$$

$$b := (0.3 \quad 0.6) \cdot h = (90 \quad 180) \cdot \text{mm} \quad b := 200 \text{ mm}$$

$$\begin{pmatrix} b_a \\ h_a \end{pmatrix} := \begin{pmatrix} b \\ h \end{pmatrix}$$

Determination of slab thickness

$$\text{Perimeter} := (L_a + L_b) \cdot 2$$

$$t_{\min} := \frac{\text{Perimeter}}{180} = 111.111 \cdot \text{mm}$$

$$\text{Assume} \quad t := 120 \text{ mm}$$

In long direction

$$b_w := b_b \quad h := h_b \quad h_f := t$$

$$h_w := h - h_f$$

$$b := \min(b_w + 2 \cdot h_w, b_w + 8 \cdot h_f) = 1.01 \text{ m}$$

$$A_1 := b_w \cdot h \quad x_1 := \frac{h}{2}$$

$$A_2 := (b - b_w) \cdot h_f \quad x_2 := \frac{h_f}{2}$$

$$x_c := \frac{x_1 \cdot A_1 + x_2 \cdot A_2}{A_1 + A_2} = 169.852 \cdot \text{mm}$$

$$I_1 := \frac{b_w \cdot h^3}{12} + A_1 \cdot (x_1 - x_c)^2$$

$$I_2 := \frac{(b - b_w) \cdot h_f^3}{12} + A_2 \cdot (x_2 - x_c)^2$$

$$I_b := I_1 + I_2 = 4.617 \times 10^5 \cdot \text{cm}^4$$

$$I_s := \frac{L_a \cdot h_f^3}{12}$$

$$w_c := 24 \frac{\text{kN}}{\text{m}^3} \quad E_c := 44 \text{MPa} \cdot \left(\frac{w_c}{\frac{\text{kN}}{\text{m}^3}} \right)^{1.5} \cdot \sqrt{\frac{f_c}{\text{MPa}}} = 2.587 \times 10^4 \cdot \text{MPa}$$

$$\alpha := \frac{E_c \cdot I_b}{E_c \cdot I_s} = 8.016 \quad \alpha_b := \alpha$$

In short direction

$$b_w := b_a \quad h := h_a \quad h_f := t$$

$$h_w := h - h_f$$

$$b := \min(b_w + 2 \cdot h_w, b_w + 8 \cdot h_f) = 0.56 \text{ m}$$

$$A_1 := b_w \cdot h \quad x_1 := \frac{h}{2}$$

$$A_2 := (b - b_w) \cdot h_f \quad x_2 := \frac{h_f}{2}$$

$$x_c := \frac{x_1 \cdot A_1 + x_2 \cdot A_2}{A_1 + A_2} = 112.326 \cdot \text{mm}$$

$$I_1 := \frac{b_w \cdot h^3}{12} + A_1 \cdot (x_1 - x_c)^2$$

$$I_2 := \frac{(b - b_w) \cdot h_f^3}{12} + A_2 \cdot (x_2 - x_c)^2$$

$$I_b := I_1 + I_2 = 7.053 \times 10^4 \cdot \text{cm}^4 \quad I_{ba} := I_b$$

$$I_s := \frac{L_b \cdot h_f^3}{12} = 8.64 \times 10^4 \cdot \text{cm}^4$$

$$\alpha := \frac{E_c \cdot I_b}{E_c \cdot I_s} = 0.816 \quad \alpha_a := \alpha$$

Required thickness of slab

$$\alpha_m := \frac{\alpha_a \cdot 2 + \alpha_b \cdot 2}{4} = 4.416$$

$$\beta := \frac{L_b}{L_a} = 1.5$$

$$L_n := L_b - 20 \text{cm} = 5.8 \text{m}$$

$$h_f := \begin{cases} \max \left[\frac{L_n \cdot \left(0.8 + \frac{f_y}{200 \text{ksi}} \right)}{36 + 5 \cdot \beta \cdot (\alpha_m - 0.2)}, 5 \text{in} \right] & \text{if } 0.2 \leq \alpha_m \leq 2.0 \\ \max \left[\frac{L_n \cdot \left(0.8 + \frac{f_y}{200 \text{ksi}} \right)}{36 + 9 \cdot \beta}, 3.5 \text{in} \right] & \text{if } 2.0 < \alpha_m \leq 5.0 \\ \text{"DDM is not applied"} & \text{otherwise} \end{cases}$$

$$h_f = 126.876 \cdot \text{mm}$$

Loads on slab

$$DL := 50 \text{mm} \cdot 22 \frac{\text{kN}}{\text{m}^3} + t \cdot 25 \frac{\text{kN}}{\text{m}^3} + 0.40 \frac{\text{kN}}{\text{m}^2} + 1.00 \frac{\text{kN}}{\text{m}^2} = 5.5 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$LL = 3 \cdot \frac{\text{kN}}{\text{m}^2}$$

$$w_u := 1.2 \cdot DL + 1.6 \cdot LL = 11.4 \cdot \frac{\text{kN}}{\text{m}^2}$$

In long direction

$$L_1 := L_b = 6 \text{ m} \quad L_n := L_1 - b_a = 5.8 \text{ m}$$

$$L_2 := L_a = 4 \text{ m} \quad \alpha_1 := \alpha_b = 8.016$$

Total static moment

$$M_0 := \frac{w_u \cdot L_2 \cdot L_n^2}{8} = 191.748 \cdot \text{kN} \cdot \text{m}$$

Longitudinal distribution of moments

$$M_{\text{neg}} := 0.65 \cdot M_0 = 124.636 \cdot \text{kN} \cdot \text{m}$$

$$M_{\text{pos}} := 0.35 \cdot M_0 = 67.112 \cdot \text{kN} \cdot \text{m}$$

Lateral distribution of moments

$$k_1 := \frac{L_2}{L_1} = 0.667 \quad k_2 := \alpha_1 \cdot \frac{L_2}{L_1} = 5.344$$

$\text{linterp2}(VX, VY, M, x, y) :=$	for $j \in 1 \dots \text{rows}(VY)$ $V_j \leftarrow \text{linterp}(VX, M^{\langle j \rangle}, x)$ $\text{linterp}(VY, V, y)$
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$$C_{\text{neg.col}} := \text{linterp2} \left[\begin{pmatrix} 0 \\ 1 \\ 10 \end{pmatrix}, \begin{pmatrix} 0.5 \\ 1.0 \\ 2.0 \end{pmatrix}, \begin{pmatrix} 0.75 & 0.75 & 0.75 \\ 0.90 & 0.75 & 0.45 \\ 0.90 & 0.75 & 0.45 \end{pmatrix}, k_2, k_1 \right] = 0.85$$

$$C_{\text{neg.mid}} := 1 - C_{\text{neg.col}} = 0.15$$

$$C_{\text{pos.col}} := \text{linterp2} \left[\begin{pmatrix} 0 \\ 1 \\ 10 \end{pmatrix}, \begin{pmatrix} 0.5 \\ 1.0 \\ 2.0 \end{pmatrix}, \begin{pmatrix} 0.60 & 0.60 & 0.60 \\ 0.90 & 0.75 & 0.45 \\ 0.90 & 0.75 & 0.45 \end{pmatrix}, k_2, k_1 \right] = 0.85$$

$$C_{\text{pos.mid}} := 1 - C_{\text{pos.col}} = 0.15$$

$$M_{\text{neg.col}} := C_{\text{neg.col}} \cdot M_{\text{neg}} = 105.941 \cdot \text{kN} \cdot \text{m}$$

$$M_{\text{neg.mid}} := C_{\text{neg.mid}} \cdot M_{\text{neg}} = 18.695 \cdot \text{kN} \cdot \text{m}$$

$$M_{\text{pos.col}} := C_{\text{pos.col}} \cdot M_{\text{pos}} = 57.045 \cdot \text{kN} \cdot \text{m}$$

$$M_{\text{pos.mid}} := C_{\text{pos.mid}} \cdot M_{\text{pos}} = 10.067 \cdot \text{kN} \cdot \text{m}$$

$$C_{\text{col.beam}} := \text{linterp} \left[\begin{pmatrix} 0 \\ 1 \\ 10 \end{pmatrix}, \begin{pmatrix} 0 \\ 0.85 \\ 0.85 \end{pmatrix}, k_2 \right] = 0.85$$

$$C_{\text{col.slab}} := 1 - C_{\text{col.beam}} = 0.15$$

$$M_{\text{neg.col.beam}} := C_{\text{col.beam}} \cdot M_{\text{neg.col}} = 90.05 \cdot \text{kN} \cdot \text{m}$$

$$M_{\text{neg.col.slab}} := C_{\text{col.slab}} \cdot M_{\text{neg.col}} = 15.891 \cdot \text{kN} \cdot \text{m}$$

$$M_{\text{pos.col.beam}} := C_{\text{col.beam}} \cdot M_{\text{pos.col}} = 48.488 \cdot \text{kN} \cdot \text{m}$$

$$M_{\text{pos.col.slab}} := C_{\text{col.slab}} \cdot M_{\text{pos.col}} = 8.557 \cdot \text{kN} \cdot \text{m}$$

$$b_{\text{col}} := \frac{\min(L_1, L_2)}{4} \cdot 2 = 2 \text{ m}$$

$$b_{\text{mid}} := L_2 - b_{\text{col}} = 2 \text{ m}$$

Top rebars in column strip

$$b := b_{\text{col}} \quad d := t - \left(20\text{mm} + 10\text{mm} + \frac{10\text{mm}}{2} \right) = 85 \cdot \text{mm}$$

$$M_u := M_{\text{neg.col.slab}} = 15.891 \cdot \text{kN} \cdot \text{m}$$

$$M_n := \frac{M_u}{0.9} = 17.657 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_n}{b \cdot d^2} = 1.222 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right) = 0.003$$

$$\rho \leq \rho_{\max} = 1$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t) = 5.489 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4}$$

$$n := \frac{A_s}{A_{s0}}$$

$$s_{\max} := \min(2 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\max}\right) = 240 \cdot \text{mm}$$

Bottom rebars in column strip

$$M_u := M_{\text{pos.col.slab}} = 8.557 \cdot \text{kN} \cdot \text{m}$$

$$M_n := \frac{M_u}{0.9} = 9.508 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_n}{b \cdot d^2} = 0.658 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right) = 0.002$$

$$\rho \leq \rho_{\max} = 1$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t) = 4.32 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4}$$

$$n := \frac{A_s}{A_{s0}}$$

$$s_{\max} := \min(2 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\max}\right) = 240 \cdot \text{mm}$$

Top rebars in middle strip

$$b := b_{\text{mid}}$$

$$M_u := M_{\text{neg.mid}} = 18.695 \cdot \text{kN} \cdot \text{m}$$

$$M_n := \frac{M_u}{0.9} = 20.773 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_n}{b \cdot d^2} = 1.438 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right) = 0.004$$

$$\rho \leq \rho_{\max} = 1$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t) = 6.494 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4} \quad n := \frac{A_s}{A_{s0}}$$

$$s_{\text{max}} := \min(2 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\text{max}}\right) = 240 \cdot \text{mm}$$

Bottom rebars in middle strip

$$M_u := M_{\text{pos.mid}} = 10.067 \cdot \text{kN} \cdot \text{m}$$

$$M_n := \frac{M_u}{0.9} = 11.185 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_n}{b \cdot d^2} = 0.774 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}}\right) = 0.002$$

$$\rho \leq \rho_{\text{max}} = 1$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t) = 4.32 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4} \quad n := \frac{A_s}{A_{s0}}$$

$$s_{\text{max}} := \min(2 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\text{max}}\right) = 240 \cdot \text{mm}$$

In short direction

$$L_1 := L_a = 4 \text{ m}$$

$$L_n := L_1 - b_b = 3.75 \text{ m}$$

$$L_2 := L_b = 6 \text{ m}$$

$$\alpha_1 := \alpha_a = 0.816$$

Total static moment

$$M_0 := \frac{w_u \cdot L_2 \cdot L_n^2}{8} = 120.234 \cdot \text{kN} \cdot \text{m}$$

Longitudinal distribution of moments

$$M_{\text{neg}} := 0.65 \cdot M_0 = 78.152 \cdot \text{kN} \cdot \text{m}$$

$$M_{\text{pos}} := 0.35 \cdot M_0 = 42.082 \cdot \text{kN} \cdot \text{m}$$

Lateral distribution of moments

$$k_1 := \frac{L_2}{L_1} = 1.5 \quad k_2 := \alpha_1 \cdot \frac{L_2}{L_1} = 1.224$$

$$C_{\text{neg.col}} := \text{linterp2} \left[\begin{pmatrix} 0 \\ 1 \\ 10 \end{pmatrix}, \begin{pmatrix} 0.5 \\ 1.0 \\ 2.0 \end{pmatrix}, \begin{pmatrix} 0.75 & 0.75 & 0.75 \\ 0.90 & 0.75 & 0.45 \\ 0.90 & 0.75 & 0.45 \end{pmatrix}, k_2, k_1 \right] = 0.6$$

$$C_{\text{neg.mid}} := 1 - C_{\text{neg.col}} = 0.4$$

$$C_{\text{pos.col}} := \text{linterp2} \left[\begin{pmatrix} 0 \\ 1 \\ 10 \end{pmatrix}, \begin{pmatrix} 0.5 \\ 1.0 \\ 2.0 \end{pmatrix}, \begin{pmatrix} 0.60 & 0.60 & 0.60 \\ 0.90 & 0.75 & 0.45 \\ 0.90 & 0.75 & 0.45 \end{pmatrix}, k_2, k_1 \right] = 0.6$$

$$C_{\text{pos.mid}} := 1 - C_{\text{pos.col}} = 0.4$$

$$M_{\text{neg.col}} := C_{\text{neg.col}} \cdot M_{\text{neg}} = 46.891 \cdot \text{kN} \cdot \text{m}$$

$$M_{\text{neg.mid}} := C_{\text{neg.mid}} \cdot M_{\text{neg}} = 31.261 \cdot \text{kN} \cdot \text{m}$$

$$M_{\text{pos.col}} := C_{\text{pos.col}} \cdot M_{\text{pos}} = 25.249 \cdot \text{kN} \cdot \text{m}$$

$$M_{\text{pos.mid}} := C_{\text{pos.mid}} \cdot M_{\text{pos}} = 16.833 \cdot \text{kN} \cdot \text{m}$$

$$C_{\text{col.beam}} := \text{linterp} \left[\begin{pmatrix} 0 \\ 1 \\ 10 \end{pmatrix}, \begin{pmatrix} 0 \\ 0.85 \\ 0.85 \end{pmatrix}, k_2 \right] = 0.85$$

$$C_{\text{col.slab}} := 1 - C_{\text{col.beam}} = 0.15$$

$$M_{\text{neg.col.beam}} := C_{\text{col.beam}} \cdot M_{\text{neg.col}} = 39.858 \cdot \text{kN} \cdot \text{m}$$

$$M_{\text{neg.col.slab}} := C_{\text{col.slab}} \cdot M_{\text{neg.col}} = 7.034 \cdot \text{kN} \cdot \text{m}$$

$$M_{\text{pos.col.beam}} := C_{\text{col.beam}} \cdot M_{\text{pos.col}} = 21.462 \cdot \text{kN} \cdot \text{m}$$

$$M_{\text{pos.col.slab}} := C_{\text{col.slab}} \cdot M_{\text{pos.col}} = 3.787 \cdot \text{kN} \cdot \text{m}$$

$$b_{\text{col}} := \frac{\min(L_1, L_2)}{4} \cdot 2 = 2 \text{ m}$$

$$b_{\text{mid}} := L_2 - b_{\text{col}} = 4 \text{ m}$$

Top rebars in column strip

$$b := b_{\text{col}}$$

$$M_u := M_{\text{neg.col.slab}} = 7.034 \cdot \text{kN} \cdot \text{m}$$

$$M_n := \frac{M_u}{0.9} = 7.815 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_n}{b \cdot d^2} = 0.541 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right) = 0.001$$

$$\rho \leq \rho_{\max} = 1$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t) = 4.32 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4} \quad n := \frac{A_s}{A_{s0}}$$

$$s_{\max} := \min(2 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\max}\right) = 240 \cdot \text{mm}$$

Bottom rebars in column strip

$$M_u := M_{\text{pos.col.slabs}} = 3.787 \cdot \text{kN} \cdot \text{m}$$

$$M_n := \frac{M_u}{0.9} = 4.208 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_n}{b \cdot d^2} = 0.291 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right) = 0.001$$

$$\rho \leq \rho_{\max} = 1$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t) = 4.32 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4} \quad n := \frac{A_s}{A_{s0}}$$

$$s_{\max} := \min(2 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\max}\right) = 240 \cdot \text{mm}$$

Top rebars in middle strip

$$b := b_{\text{mid}}$$

$$M_u := M_{\text{neg.mid}} = 31.261 \cdot \text{kN} \cdot \text{m}$$

$$M_n := \frac{M_u}{0.9} = 34.734 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_n}{b \cdot d^2} = 1.202 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f'_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f'_c}} \right) = 0.003$$

$$\rho \leq \rho_{\max} = 1$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t) = 10.792 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4} \quad n := \frac{A_s}{A_{s0}} \quad s_{\max} := \min(2 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\max}\right) = 240 \cdot \text{mm}$$

Bottom rebars in middle strip

$$M_u := M_{\text{pos.mid}} = 16.833 \cdot \text{kN} \cdot \text{m}$$

$$M_n := \frac{M_u}{0.9} = 18.703 \cdot \text{kN} \cdot \text{m}$$

$$R := \frac{M_n}{b \cdot d^2} = 0.647 \cdot \text{MPa}$$

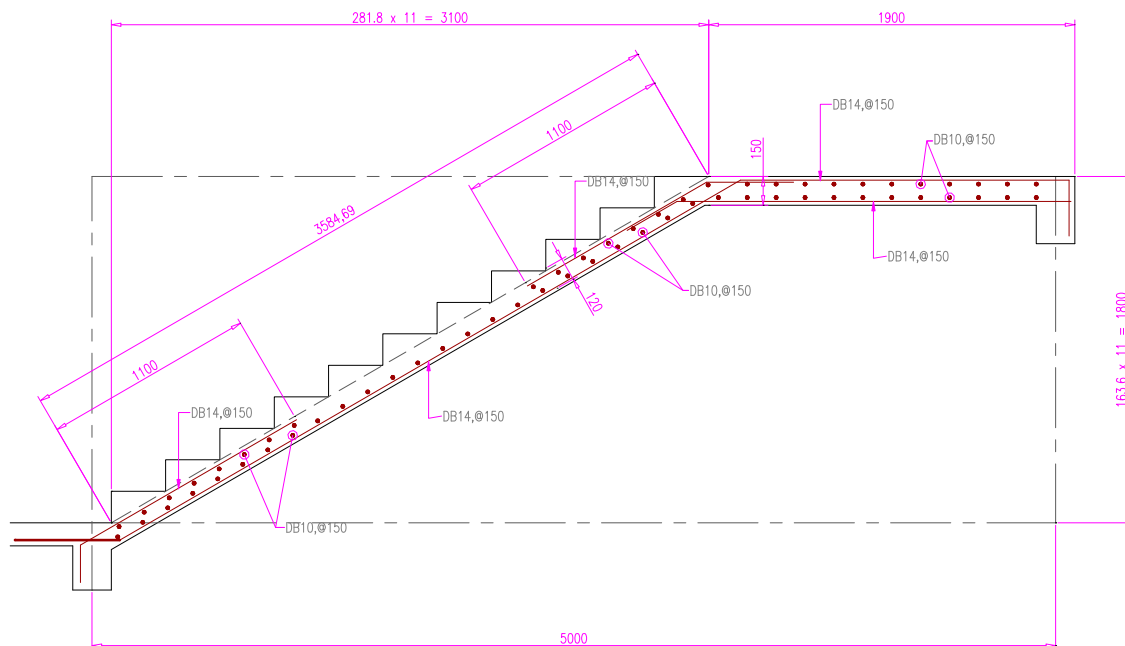
$$\rho := 0.85 \cdot \frac{f_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f_c}}\right) = 0.002 \quad \rho \leq \rho_{\max} = 1$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t) = 8.64 \cdot \text{cm}^2$$

$$A_{s0} := \frac{\pi \cdot (10\text{mm})^2}{4} \quad n := \frac{A_s}{A_{s0}} \quad s_{\max} := \min(2 \cdot t, 450\text{mm})$$

$$s := \min\left(\text{Floor}\left(\frac{b}{n}, 10\text{mm}\right), s_{\max}\right) = 240 \cdot \text{mm}$$

18. Design of Staircase



Step dimension

$$G := \frac{3.1\text{m}}{11} = 281.818 \cdot \text{mm}$$

$$H := \frac{1.8\text{m}}{11} = 163.636 \cdot \text{mm}$$

$$G + 2 \cdot H = 60.909 \cdot \text{cm}$$

Reference:

$$G + 2 \cdot H = 60\text{cm} \dots 64\text{cm}$$

$$H = 150\text{mm} \dots 190\text{mm}$$

Number of steps

$$n := 11$$

Loads on waist slab

Slope angle

$$\alpha := \text{atan}\left(\frac{H}{G}\right) = 30.141 \cdot \text{deg}$$

Thickness of waist slab

$$t_{\text{waist}} := 120\text{mm}$$

Step cover

$$\text{Cover} := 50\text{mm} \cdot (H + G) \cdot 22 \frac{\text{kN}}{\text{m}^3} \cdot \frac{1\text{m}}{1\text{m} \cdot G} = 1.739 \cdot \frac{\text{kN}}{\text{m}^2}$$

Concrete step

$$\text{Step} := \frac{G \cdot H}{2} \cdot 24 \frac{\text{kN}}{\text{m}^3} \cdot \frac{1\text{m}}{1\text{m} \cdot G} = 1.964 \cdot \frac{\text{kN}}{\text{m}^2}$$

RC slab

$$\text{Slab} := t_{\text{waist}} \cdot 25 \frac{\text{kN}}{\text{m}^3} \cdot \frac{1\text{m}^2}{1\text{m}^2 \cdot \cos(\alpha)} = 3.469 \cdot \frac{\text{kN}}{\text{m}^2}$$

Ceiling	$\text{Ceiling} := 0.40 \frac{\text{kN}}{\text{m}^2} \cdot \frac{1\text{m}^2}{1\text{m}^2 \cdot \cos(\alpha)} = 0.463 \cdot \frac{\text{kN}}{\text{m}^2}$
Handrail	$\text{Handrail} := 0.50 \frac{\text{kN}}{\text{m}^2}$
Dead load	$\text{DL} := \text{Cover} + \text{Step} + \text{Slab} + \text{Ceiling} + \text{Handrail}$
	$\text{DL} = 8.134 \cdot \frac{\text{kN}}{\text{m}^2}$
Live load	$\text{LL} := 4.80 \frac{\text{kN}}{\text{m}^2}$
Factored load	$w_{\text{waist}} := 1.2 \cdot \text{DL} + 1.6 \cdot \text{LL} = 17.441 \cdot \frac{\text{kN}}{\text{m}^2}$

Loads on landing slab

Thickness of landing slab	$t_{\text{landing}} := 150\text{mm}$
Slab cover	$\text{Cover} := 50\text{mm} \cdot 22 \frac{\text{kN}}{\text{m}^3} = 1.1 \cdot \frac{\text{kN}}{\text{m}^2}$
RC slab	$\text{Slab} := t_{\text{landing}} \cdot 25 \frac{\text{kN}}{\text{m}^3} = 3.75 \cdot \frac{\text{kN}}{\text{m}^2}$
Ceiling	$\text{Ceiling} := 0.40 \frac{\text{kN}}{\text{m}^2}$
Handrail	$\text{Handrail} := 0.50 \frac{\text{kN}}{\text{m}^2}$
Dead load	$\text{DL} := \text{Cover} + \text{Slab} + \text{Ceiling} + \text{Handrail} = 5.75 \cdot \frac{\text{kN}}{\text{m}^2}$
Live load	$\text{LL} := 4.80 \frac{\text{kN}}{\text{m}^2}$
Factored load	$w_{\text{landing}} := 1.2 \cdot \text{DL} + 1.6 \cdot \text{LL} = 14.58 \cdot \frac{\text{kN}}{\text{m}^2}$

Analysis of Staircase

Concrete modulus of elasticity

$$f'_c := 25\text{MPa}$$

$$w_c := 24 \frac{\text{kN}}{\text{m}^3}$$

$$E_c := 44\text{MPa} \cdot \left(\frac{w_c}{\frac{\text{kN}}{\text{m}^3}} \right)^{1.5} \cdot \sqrt{\frac{f'_c}{\text{MPa}}} = 2.587 \times 10^4 \cdot \text{MPa}$$

Geometry of staircase

$$L_0 := 3.1\text{m} \quad L_2 := 1.9\text{m} \quad h := 1.8\text{m}$$

$$L_1 := \sqrt{L_0^2 + h^2} = 3.585\text{m}$$

$$t_1 := t_{\text{waist}} = 120\cdot\text{mm} \quad t_2 := t_{\text{landing}} = 150\cdot\text{mm}$$

Flexural stiffness

$$b := 1\text{m}$$

$$EI_1 := E_c \cdot \frac{b \cdot t_1^3}{12} \quad EI_2 := E_c \cdot \frac{b \cdot t_2^3}{12}$$

Loads on staircase

$$w_1 := w_{\text{waist}} \cdot b = 17.441 \cdot \frac{\text{kN}}{\text{m}}$$

$$w_2 := w_{\text{landing}} \cdot b = 14.58 \cdot \frac{\text{kN}}{\text{m}}$$

Coefficients

$$r_{11} := 4 \cdot \frac{EI_1}{L_1} + 3 \cdot \frac{EI_2}{L_2} \quad R_{1p} := \frac{w_1 \cdot L_0^2}{12} - \frac{w_2 \cdot L_2^2}{8}$$

Angular rotation

$$Z_1 := \frac{-R_{1p}}{r_{11}} = -4.723 \times 10^{-4} \quad \varphi_B := Z_1$$

Bending moments

$$M_A := 2 \cdot \frac{EI_1}{L_1} \cdot Z_1 - \frac{w_1 \cdot L_0^2}{12} = -14.949 \cdot \text{kN} \cdot \text{m}$$

$$M_{BA} := -4 \cdot \frac{EI_1}{L_1} \cdot Z_1 - \frac{w_1 \cdot L_0^2}{12} = -12.004 \cdot \text{kN} \cdot \text{m}$$

$$M_{BC} := 3 \cdot \frac{EI_2}{L_2} \cdot Z_1 - \frac{w_2 \cdot L_2^2}{8} = -12.004 \cdot \text{kN} \cdot \text{m}$$

$$M_C := 0$$

Shears

$$w_0 := w_1 \cdot \cos(\alpha)^2 = 13.043 \cdot \frac{\text{kN}}{\text{m}}$$

$$V_{AB} := \frac{M_{BA} - M_A}{L_1} + \frac{w_0 \cdot L_1}{2} = 24.199 \cdot \text{kN}$$

$$V_{BA} := V_{AB} - w_0 \cdot L_1 = -22.557 \cdot \text{kN}$$

$$V_{BC} := \frac{M_C - M_{BC}}{L_2} + \frac{w_2 \cdot L_2}{2} = 20.169 \cdot \text{kN}$$

$$V_{CB} := V_{BC} - w_2 \cdot L_2 = -7.533 \cdot \text{kN}$$

Positive moments

$$x_1 := \frac{V_{AB}}{w_0} = 1.855 \text{ m}$$

$$M_{\max, AB} := M_A + V_{AB} \cdot x_1 - \frac{w_0 \cdot x_1^2}{2} = 7.5 \cdot \text{kN} \cdot \text{m}$$

$$x_2 := \frac{V_{BC}}{w_2} = 1.383 \text{ m}$$

$$M_{\max, BC} := M_{BC} + V_{BC} \cdot x_2 - \frac{w_2 \cdot x_2^2}{2} = 1.946 \cdot \text{kN} \cdot \text{m}$$

Design of Staircase

Materials

$$f'_c := 25 \text{ MPa} \quad f_y := 390 \text{ MPa}$$

$$\epsilon_u := 0.003$$

$$\beta_1 := 0.65 \max \left[\left(0.85 - 0.05 \cdot \frac{f'_c - 27.6 \text{ MPa}}{6.9 \text{ MPa}} \right) \min 0.85 \right] = 0.85$$

$$\rho_{\max} := 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + 0.005} = 0.017$$

$$\rho_{\min} := \max \left(\frac{0.249 \text{ MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}}}{f_y}, \frac{1.379 \text{ MPa}}{f_y} \right)$$

$$\rho_{\text{shrinkage}} := \begin{cases} \text{return } 0.0020 & \text{if } f_y \leq 50 \text{ ksi} \\ \text{return } 0.0018 & \text{if } f_y \leq 60 \text{ ksi} \\ \text{return } \max \left(0.0018 \cdot \frac{60 \text{ ksi}}{f_y}, 0.0014 \right) & \text{otherwise} \end{cases} \quad \rho_{\text{shrinkage}} = 0.0018$$

Top rebars in waist slab

$$b = 1 \text{ m} \quad d := t_{\text{waist}} - \left(30\text{mm} + \frac{16\text{mm}}{2} \right) = 82 \cdot \text{mm}$$

$$M_u := \max(|M_A|, |M_{BA}|) = 14.949 \cdot \text{kN} \cdot \text{m} \quad M_n := \frac{M_u}{0.9}$$

$$R := \frac{M_n}{b \cdot d^2} = 2.47 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f_c}} \right) = 0.00675$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t_{\text{waist}}) = 5.537 \cdot \text{cm}^2$$

$$\frac{b}{200\text{mm}} \cdot \frac{\pi \cdot (14\text{mm})^2}{4} = 7.697 \cdot \text{cm}^2$$

Top rebars in landing slab

$$b = 1 \text{ m} \quad d := t_{\text{landing}} - \left(30\text{mm} + \frac{16\text{mm}}{2} \right) = 112 \cdot \text{mm}$$

$$M_u := \max(|M_{BC}|, |M_C|) = 12.004 \cdot \text{kN} \cdot \text{m} \quad M_n := \frac{M_u}{0.9}$$

$$R := \frac{M_n}{b \cdot d^2} = 1.063 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f_c}} \right) = 0.0028$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t_{\text{waist}}) = 3.134 \cdot \text{cm}^2$$

$$\frac{b}{200\text{mm}} \cdot \frac{\pi \cdot (10\text{mm})^2}{4} = 3.927 \cdot \text{cm}^2$$

Bottom rebars in waist slab

$$b = 1 \text{ m} \quad d := t_{\text{waist}} - \left(30\text{mm} + \frac{16\text{mm}}{2} \right) = 82 \cdot \text{mm}$$

$$M_u := M_{\text{max.AB}} = 7.5 \cdot \text{kN} \cdot \text{m} \quad M_n := \frac{M_u}{0.9}$$

$$R := \frac{M_n}{b \cdot d^2} = 1.239 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f_c}} \right) = 0.00328$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t_{\text{waist}}) = 2.687 \cdot \text{cm}^2$$

$$\frac{b}{200\text{mm}} \cdot \frac{\pi \cdot (10\text{mm})^2}{4} = 3.927 \cdot \text{cm}^2$$

Bottom rebars in landing slab

$$b = 1\text{ m} \quad d := t_{\text{landing}} - \left(30\text{mm} + \frac{16\text{mm}}{2} \right) = 112 \cdot \text{mm}$$

$$M_u := M_{\text{max.BC}} = 1.946 \cdot \text{kN} \cdot \text{m} \quad M_n := \frac{M_u}{0.9}$$

$$R := \frac{M_n}{b \cdot d^2} = 0.172 \cdot \text{MPa}$$

$$\rho := 0.85 \cdot \frac{f_c}{f_y} \cdot \left(1 - \sqrt{1 - 2 \cdot \frac{R}{0.85 \cdot f_c}} \right) = 0.00044$$

$$A_s := \max(\rho \cdot b \cdot d, \rho_{\text{shrinkage}} \cdot b \cdot t_{\text{waist}}) = 2.16 \cdot \text{cm}^2$$

$$\frac{b}{200\text{mm}} \cdot \frac{\pi \cdot (10\text{mm})^2}{4} = 3.927 \cdot \text{cm}^2$$

Link rebars

$$b := 1\text{ m} \quad t := \max(t_{\text{waist}}, t_{\text{landing}}) = 150 \cdot \text{mm}$$

$$A_s := \rho_{\text{shrinkage}} \cdot b \cdot t = 2.7 \cdot \text{cm}^2$$

$$\frac{b}{250\text{mm}} \cdot \frac{\pi \cdot (10\text{mm})^2}{4} = 3.142 \cdot \text{cm}^2$$

19. Deflection

A. Immediate (Initial) Deflection

Initial or short-term deflection in midspan of continuous beam

$$\Delta_i = K \cdot \frac{5 \cdot M_a \cdot L_n^2}{48 \cdot E_c \cdot I_e}$$

where M_a = support moment for cantilever or midspan moment for simple or continuous beam

L_n = span length of beam

E_c = concrete modulus of elasticity

I_e = effective moment of inertia of cracked section

K = deflection coefficient for uniform distributed load w

1. Cantilever $K = 2.40$

2. Simple beam $K = 1.0$

3. Continuous beam $K = 1.2 - 0.2 \frac{M_0}{M_a}$

4. Fixed-hinged beam

Midspan deflection $K = 0.8$

Max. deflection using max. moment $K = 0.74$

5. Fixed-fixed beam $K = 0.60$

where $M_0 = \frac{w \cdot L_n^2}{8}$

Effective moment of inertia of cracked section

$$I_e = I_{cr} + \left(\frac{M_{cr}}{M_a} \right)^3 \cdot (I_g - I_{cr}) \leq I_g$$

where

I_{cr} = moment of inertia of cracked transformed section

I_g = moment of inertia of gross section

M_{cr} = cracking moment

Case of continuous beams

According ACI Code 9.5.2 for continuous span

$$I_e = 0.50 \cdot I_m + 0.25 \cdot (I_{e1} + I_{e2})$$

Beams with both ends continuous

$$I_e = 0.70 \cdot I_m + 0.15 \cdot (I_{e1} + I_{e2})$$

Beams with one end continuous

$$I_e = 0.85 \cdot I_m + 0.15 \cdot I_{\text{cont.end}}$$

where

I_m = midspan section I_e

I_{e1}, I_{e2} = I_e for the respective beam ends

$I_{\text{cont.end}}$ = I_e of continuous end

Transformed Section

$$\epsilon_c = \frac{f_c}{E_c} = \epsilon_s = \frac{f_s}{E_s}$$

From which $f_s = \frac{E_s}{E_c} \cdot f_c = n \cdot f_c$

where $n = \frac{E_s}{E_c}$ is a modulus ratio

Axial force

$$P = A_c \cdot f_c + A_s \cdot f_s = (A_c + n \cdot A_s) \cdot f_c = A_t \cdot f_c$$

where $A_t = A_c + n \cdot A_s = A_g + (n - 1) \cdot A_s$

is a transformed section

A_g = area of gross section

Cracking Moment

$$M_{cr} = \frac{I_{ut} \cdot f_t}{y_t} \quad (\text{exact expression})$$

$$M_{cr} = \frac{I_g \cdot f_r}{y_t} \quad (\text{simplified expression})$$

where

I_{ut} = moment of inertia of uncracked transformed section A_t

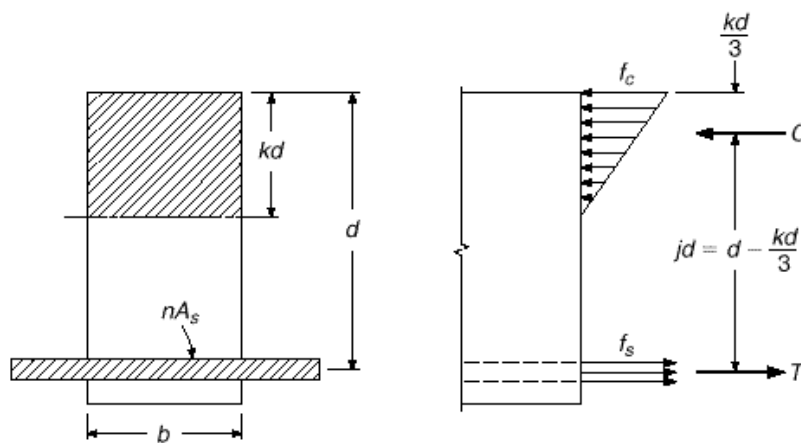
I_g = moment of inertia of gross section A_g

f_r = modulus of rupture

$$f_r = 7.5 \text{psi} \cdot \sqrt{\frac{f'_c}{\text{psi}}} = 0.623 \text{MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}}$$

y_t = distance from neutral axis to the tension face

Moment of inertia of cracked section



Condition of strain compatibility

$$\frac{\epsilon_s}{\epsilon_u} = \frac{d - x}{x} \quad \epsilon_s = \epsilon_u \cdot \frac{d - x}{x}$$

Equilibrium in forces

$$C = T$$

$$\frac{f_c \cdot x}{2} \cdot b = A_s \cdot f_s$$

$$\frac{E_c \cdot \epsilon_u \cdot b \cdot x}{2} = A_s \cdot E_s \cdot \epsilon_s = A_s \cdot E_s \cdot \epsilon_u \cdot \frac{d - x}{x}$$

$$b \cdot x^2 = 2 \cdot n \cdot A_s \cdot (d - x)$$

$$\frac{b \cdot x^2}{b \cdot d^2} = \frac{2 \cdot n \cdot A_s \cdot (d - x)}{b \cdot d^2}$$

$$\left(\frac{x}{d}\right)^2 = 2 \cdot n \cdot \rho \cdot \left(1 - \frac{x}{d}\right)$$

$$\left(\frac{x}{d}\right)^2 + 2 \cdot n \cdot \rho \cdot \frac{x}{d} - 2 \cdot n \cdot \rho = 0$$

$$\frac{x}{d} = -n \cdot \rho + \sqrt{(n \cdot \rho)^2 + 2 \cdot n \cdot \rho}$$

Moment of inertia of cracked section

$$I_{cr} = \frac{b \cdot x^3}{3} + n \cdot A_s \cdot (d - x)^2$$

B. Long-Term Deflection

Long-term deflection due to combined effect of creep and shrinkage

$$\Delta_t = \lambda \cdot \Delta_i \quad \lambda = \frac{\xi}{1 + 50 \cdot \rho'}$$

where $\rho' = \frac{A'_s}{b \cdot d}$

ξ = time-dependent coefficient

Sustained load duration	Value ξ
5 year and more	2.0
12 months	1.4
6 months	1.2
3 months	1.0

C. Minimum Depth-Span Ratio

TABLE 9.5(a)—MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE CALCULATED

	Minimum thickness, h			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

Notes:

Values given shall be used directly for members with normalweight concrete (density $w_c = 2320 \text{ kg/m}^3$) and Grade 420 reinforcement. For other conditions, the values shall be modified as follows:

a) For structural lightweight concrete having unit density, w_c , in the range 1440–1920 kg/m^3 , the values shall be multiplied by $(1.65 - 0.003w_c)$ but not less than 1.09.

b) For f_y other than 420 MPa, the values shall be multiplied by $(0.4 + f_y/700)$.

D. Permissible Deflection

TABLE 9.5(b) — MAXIMUM PERMISSIBLE COMPUTED DEFLECTIONS

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$\ell/180^*$
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$\ell/360$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load) [†]	$\ell/480^\ddagger$
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\ell/240^\S$

* Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

† Long-term deflection shall be determined in accordance with 9.5.2.5 or 9.5.4.3, but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

‡ Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.

§ Limit shall not be greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.

Example

Span length	$L_n := 9.8\text{m} - 60\text{cm} = 9.2\text{m}$	
Beam section	$b := 300\text{mm}$	$h := 750\text{mm}$
Bending moments	$M_{D,\text{neg}} := 419.34\text{kN}\cdot\text{m}$	$M_{D,\text{pos}} := 319.33\text{kN}\cdot\text{m}$
	$M_{L,\text{neg}} := 223.09\text{kN}\cdot\text{m}$	$M_{L,\text{pos}} := 176.58\text{kN}\cdot\text{m}$
Steel re-bars	$A_{s,\text{sup}} := 6 \cdot \frac{\pi \cdot (25\text{mm})^2}{4} = 29.452 \cdot \text{cm}^2$	
	$A'_{s,\text{sup}} := 3 \cdot \frac{\pi \cdot (25\text{mm})^2}{4} = 14.726 \cdot \text{cm}^2$	
	$A_{s,\text{mid}} := 5 \cdot \frac{\pi \cdot (25\text{mm})^2}{4} = 24.544 \cdot \text{cm}^2$	
	$A'_{s,\text{mid}} := 3 \cdot \frac{\pi \cdot (25\text{mm})^2}{4} = 14.726 \cdot \text{cm}^2$	
Materials	$f'_c := 25\text{MPa}$	
	$f_y := 390\text{MPa}$	

Solution

Modulus of rupture

$$f_r := 0.623\text{MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}} = 3.115 \cdot \text{MPa}$$

Modulus ratio

$$E_s := 2 \cdot 10^5 \text{MPa}$$

$$w_c := 24 \frac{\text{kN}}{\text{m}^3}$$

$$E_c := 44\text{MPa} \cdot \left(\frac{w_c}{\frac{\text{kN}}{\text{m}^3}} \right)^{1.5} \cdot \sqrt{\frac{f'_c}{\text{MPa}}} = 2.587 \times 10^4 \cdot \text{MPa}$$

$$n := \frac{E_s}{E_c} = 7.732$$

Support section

Centroid of uncracked section

$$A_1 := b \cdot h \quad y_1 := \frac{h}{2}$$

$$A_s := A_{s,\text{sup}} \quad d := h - \left(30\text{mm} + 10\text{mm} + 20\text{mm} + 25\text{mm} + \frac{40\text{mm}}{2} \right) = 645 \cdot \text{mm}$$

$$A_2 := n \cdot A_s \quad y_2 := d$$

$$y_c := \frac{A_1 \cdot y_1 + A_2 \cdot y_2}{A_1 + A_2} = 399.815 \cdot \text{mm}$$

Moment of inertia of gross section

$$I_1 := \frac{b \cdot h^3}{12}$$

$$I_g := I_1 + A_1 \cdot (y_1 - y_c)^2 + A_2 \cdot (y_2 - y_c)^2 = 1.205 \times 10^6 \cdot \text{cm}^4$$

Cracking moment

$$y_t := h - y_c = 350.185 \cdot \text{mm}$$

$$M_{cr} := f_r \cdot \frac{I_g}{y_t} = 107.228 \cdot \text{kN} \cdot \text{m}$$

Location of neutral axis of cracked section

$$C = T$$

$$\frac{f_c \cdot x}{2} \cdot b = A_s \cdot f_s$$

$$E_c \cdot \epsilon_u \cdot x \cdot b = 2 \cdot A_s \cdot E_s \cdot \epsilon_s$$

$$\epsilon_u \cdot x \cdot b = 2 \cdot A_s \cdot \frac{E_s}{E_u} \cdot \epsilon_u \cdot \frac{d - x}{x}$$

$$b \cdot x^2 = 2 \cdot A_s \cdot n \cdot (d - x)$$

$$\left(\frac{x}{d}\right)^2 = 2 \cdot \rho \cdot n \cdot \left(1 - \frac{x}{d}\right) \quad \rho := \frac{A_s}{b \cdot d} = 0.015$$

$$\left(\frac{x}{d}\right)^2 + 2 \cdot \rho \cdot n \cdot \frac{x}{d} - 2 \cdot \rho \cdot n = 0$$

$$x := d \cdot \left[-\rho \cdot n + \sqrt{(\rho \cdot n)^2 + 2 \cdot \rho \cdot n} \right] = 246.092 \cdot \text{mm}$$

Moment of inertia of cracked section

$$I_{cr} := \frac{b \cdot x^3}{3} + A_2 \cdot (d - x)^2 = 5.114 \times 10^5 \cdot \text{cm}^4$$

Effective moment of inertia of cracked section

$$M_{neg} := M_{D,neg} + M_{L,neg} = 642.43 \cdot \text{kN} \cdot \text{m} \quad M_a := M_{neg}$$

$$I_{e1} := \min \left[I_{cr} + \left(\frac{M_{cr}}{M_a} \right)^3 \cdot (I_g - I_{cr}), I_g \right] = 5.146 \times 10^5 \cdot \text{cm}^4$$

$$I_{e2} := I_{e1}$$

Midspan section

Centroid of uncracked section

$$A_1 := b \cdot h \quad y_1 := \frac{h}{2}$$

$$A_s := A_{s,\text{mid}} \quad d := h - \left(30\text{mm} + 10\text{mm} + 25\text{mm} + \frac{40\text{mm}}{2} \right) = 665\text{mm}$$

$$A_2 := n \cdot A_s \quad y_2 := d$$

$$y_c := \frac{A_1 \cdot y_1 + A_2 \cdot y_2}{A_1 + A_2} = 397.557\text{mm}$$

Moment of inertia of gross section

$$I_1 := \frac{b \cdot h^3}{12}$$

$$I_g := I_1 + A_1 \cdot (y_1 - y_c)^2 + A_2 \cdot (y_2 - y_c)^2 = 1.202 \times 10^6 \cdot \text{cm}^4$$

Cracking moment

$$y_t := h - y_c = 352.443\text{mm}$$

$$M_{cr} := f_r \cdot \frac{I_g}{y_t} = 106.225\text{ kN}\cdot\text{m}$$

Location of neutral axis of cracked section

$$C = T$$

$$\frac{f_c \cdot x}{2} \cdot b = A_s \cdot f_s$$

$$E_c \cdot \epsilon_u \cdot x \cdot b = 2 \cdot A_s \cdot E_s \cdot \epsilon_s$$

$$\epsilon_u \cdot x \cdot b = 2 \cdot A_s \cdot \frac{E_s}{E_u} \cdot \epsilon_u \cdot \frac{d - x}{x}$$

$$b \cdot x^2 = 2 \cdot A_s \cdot n \cdot (d - x)$$

$$\left(\frac{x}{d} \right)^2 = 2 \cdot \rho \cdot n \cdot \left(1 - \frac{x}{d} \right) \quad \rho := \frac{A_s}{b \cdot d} = 0.012$$

$$\left(\frac{x}{d} \right)^2 + 2 \cdot \rho \cdot n \cdot \frac{x}{d} - 2 \cdot \rho \cdot n = 0$$

$$x := d \cdot \left[-\rho \cdot n + \sqrt{(\rho \cdot n)^2 + 2 \cdot \rho \cdot n} \right] = 233.616\text{mm}$$

Moment of inertia of cracked section

$$I_{cr} := \frac{b \cdot x^3}{3} + A_2 \cdot (d - x)^2 = 4.806 \times 10^5 \cdot \text{cm}^4$$

Effective moment of inertia of cracked section

$$M_{pos} := M_{D,pos} + M_{L,pos} = 495.91 \cdot \text{kN} \cdot \text{m} \quad M_a := M_{pos}$$

$$I_m := \min \left[I_{cr} + \left(\frac{M_{cr}}{M_a} \right)^3 \cdot (I_g - I_{cr}), I_g \right] = 4.877 \times 10^5 \cdot \text{cm}^4$$

Calculation of deflection

Effective moment of inertia

$$I_e := 0.70 \cdot I_m + 0.15 \cdot (I_{e1} + I_{e2}) = 4.958 \times 10^5 \cdot \text{cm}^4$$

Initial deflection due to dead and live loads

$$M_a = 495.91 \cdot \text{kN} \cdot \text{m}$$

$$M_0 := M_{neg} + M_{pos} = 1138.34 \cdot \text{kN} \cdot \text{m}$$

$$K := 1.2 - 0.2 \frac{M_0}{M_a} = 0.741$$

$$\Delta_{D+L} := K \cdot \frac{5 \cdot M_a \cdot L_n^2}{48 \cdot E_c \cdot I_e} = 25.259 \cdot \text{mm}$$

Long-term deflection due to dead load

$$\xi := 2$$

$$A'_s := A'_{s,mid} \quad \rho' := \frac{A'_s}{b \cdot d}$$

$$\lambda := \frac{\xi}{1 + 50 \cdot \rho'} = 1.461$$

$$\Delta_D := \lambda \cdot \Delta_{D+L} \cdot \frac{M_{D,pos}}{M_{pos}} = 23.761 \cdot \text{mm}$$

Long-term deflection due to sustained live load

$$\Delta_{0.20L} := \Delta_{D+L} \cdot \frac{0.20 \cdot M_{L,pos}}{M_{pos}} = 1.799 \cdot \text{mm}$$

Short-term deflection due to live load

$$\Delta_{0.80L} := \Delta_{D+L} \cdot \frac{0.80 \cdot M_{L,pos}}{M_{pos}} = 7.195 \cdot \text{mm}$$

Total deflection

$$\Delta := \Delta_D + \Delta_{0.20L} + \Delta_{0.80L} = 32.755 \cdot \text{mm}$$

Permissible deflection

$$\frac{L_n}{480} = 19.167 \cdot \text{mm}$$

$$\frac{L_n}{360} = 25.556 \cdot \text{mm}$$

20. Development Lengths

A. Development length of deformed bar in tension

Diameter of deformed bar	$d_b := 20\text{mm}$
Steel yield strength	$f_y := 390\text{MPa}$
Concrete compression strength	$f'_c := 25\text{MPa}$
Depth of concrete below development length	$H := 350\text{mm}$

Reinforcement coating

Epoxy Coated
Uncoated

Type of concrete

Lightweight
Normalweight

Concrete cover $c := 1 \cdot d_b$

Clear spacing of re-bars $s := 2 \cdot d_b$

$$\psi_t := \begin{cases} 1.3 & \text{if } H > 300\text{mm} \\ 1.0 & \text{otherwise} \end{cases} \quad \psi_t = 1.3$$

$$\psi_e := \begin{cases} 1.0 & \text{if Coating} = \text{"Uncoated"} \\ \text{otherwise} \\ \begin{cases} 1.5 & \text{if } c < 3d_b \vee s < 6d_b \\ 1.2 & \text{otherwise} \end{cases} \end{cases} \quad \psi_e = 1$$

$$\psi_s := \begin{cases} 0.8 & \text{if } d_b \leq 20\text{mm} \\ 1.0 & \text{otherwise} \end{cases} \quad \psi_s = 0.8$$

$$\lambda := \begin{cases} 1.3 & \text{if Concrete} = \text{"Lightweight"} \\ 1.0 & \text{otherwise} \end{cases} \quad \lambda = 1$$

$K_{tr} := 0$ (for a design simplification)

$$c_b := (1.5d_b) \max \left[\min \left(c + \frac{d_b}{2}, \frac{s + d_b}{2} \right), \min(2.5d_b) \right] = 30 \cdot \text{mm} \quad \frac{c_b + K_{tr}}{d_b} = 1.5$$

Development of tension bar in tension

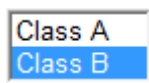
$$L_d := \max \left[\frac{f_y}{1.107 \text{ MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}}} \cdot \frac{\psi_t \cdot \psi_e \cdot \psi_s \cdot \lambda}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \cdot d_b, 300 \text{ mm} \right] = 977.055 \cdot \text{mm}$$

$$\frac{L_d}{d_b} = 48.853$$

$$\text{Ceil}(L_d, 10 \text{ mm}) = 980 \cdot \text{mm}$$

B. Splice length in tension

Splice class



$$L_{st} := \begin{cases} 1.0 \cdot L_d & \text{if Class} = \text{"Class A"} \\ 1.3 \cdot L_d & \text{otherwise} \end{cases}$$

$$L_{st} = 1270.172 \cdot \text{mm} \quad \frac{L_{st}}{d_b} = 63.509$$

$$\text{Ceil}(L_{st}, 10 \text{ mm}) = 1280 \cdot \text{mm}$$

C. Development length of deformed bar in compression

Diameter of development bar

$$d_b := 32 \text{ mm}$$

Steel yield strength

$$f_y := 390 \text{ MPa}$$

Concrete compression strength

$$f'_c := 25 \text{ MPa}$$

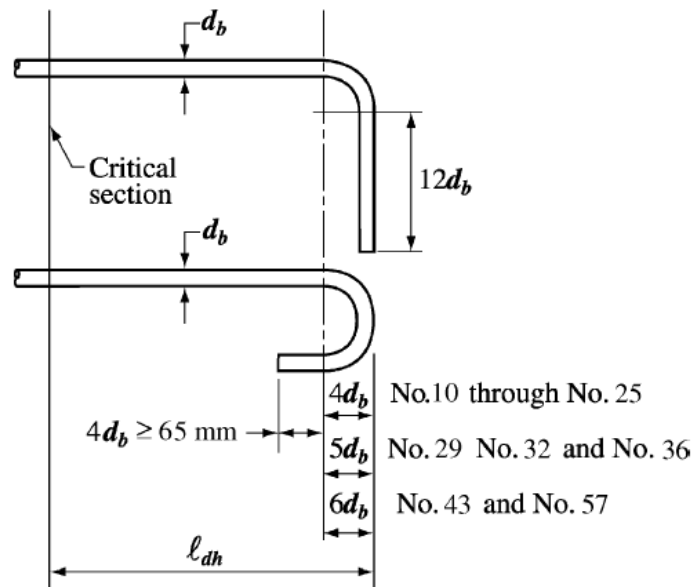
Development length in compression

$$L_{dc} := \max \left(\frac{f_y}{4.152 \text{ MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}}} \cdot d_b, \frac{f_y}{22.983 \text{ MPa}} \cdot d_b, 200 \text{ mm} \right) = 601.156 \cdot \text{mm}$$

$$\frac{L_{dc}}{d_b} = 18.786$$

$$\text{Ceil}(L_{dc}, 10 \text{ mm}) = 610 \cdot \text{mm}$$

D. Development length of standard hook in tension



Diameter of development bar $d_b := 10\text{mm}$

Steel yield strength $f_y := 390\text{MPa}$

Concrete compression strength $f'_c := 25\text{MPa}$

Reinforcement coating

Epoxy Coated
Uncoated

Type of concrete

Lightweight
Normalweight

Side cover $c_{\text{side}} := 65\text{mm}$

Cover beyond hook $c_{\text{beyond}} := 50\text{mm}$

$\psi_e := \begin{cases} 1.0 & \text{if Coating} = \text{"Uncoated"} \\ \text{otherwise} & \begin{cases} 1.5 & \text{if } c < 3d_b \vee s < 6d_b \\ 1.2 & \text{otherwise} \end{cases} \end{cases}$ $\psi_e = 1$

$\lambda := \begin{cases} 1.3 & \text{if Concrete} = \text{"Lightweight"} \\ 1.0 & \text{otherwise} \end{cases}$ $\lambda = 1$

Modified factor $\alpha := \begin{cases} 0.8 & \text{if } (c_{\text{side}} < 65\text{mm}) \vee (c_{\text{beyond}} < 50\text{mm}) \\ 0.7 & \text{otherwise} \end{cases}$

$$\alpha = 0.7$$

Development length of standard hook in tension

$$L_{dh} := \alpha \cdot \max \left(\frac{\psi_e \cdot \lambda \cdot f_y}{4.152 \text{MPa} \cdot \sqrt{\frac{f'_c}{\text{MPa}}}} \cdot d_b, 8d_b, 150\text{mm} \right) = 131.503 \cdot \text{mm}$$

$$\frac{L_{dh}}{d_b} = 13.15$$

$$\text{Ceil}(L_{dh}, 10\text{mm}) = 140 \cdot \text{mm}$$

E. Lap Splice Length in Compression

Diameter of splice bar

$$d_b := 25\text{mm}$$

Steel yield strength

$$f_y := 390\text{MPa}$$

Lap splice length in compression

$$L_{sc} := \begin{cases} \max \left(\frac{f_y}{13.79\text{MPa}} \cdot d_b, 300\text{mm} \right) & \text{if } f_y \leq 60\text{ksi} \\ \max \left[\left(\frac{f_y}{7.661\text{MPa}} - 24 \right) \cdot d_b, 300\text{mm} \right] & \text{otherwise} \end{cases}$$

$$L_{sc} = 707.034 \cdot \text{mm}$$

$$\text{Ceil}(L_{sc}, 10\text{mm}) = 710 \cdot \text{mm}$$

$$\frac{L_{sc}}{d_b} = 28.281$$

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